BSSS

AUSTRALIAN CAPITAL TERRITORY

Mathematical Methods

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Cover Art provided by Canberra College student Aidan Giddings

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The ACT Senior Secondary System

The ACT senior secondary system recognises a range of university, vocational or life skills pathways.

The system is based on the premise that teachers are experts in their area: they know their students and community and are thus best placed to develop curriculum and assess students according to their needs and interests. Students have ownership of their learning and are respected as young adults who have a voice.

A defining feature of the system is school-based curriculum and continuous assessment. School-based curriculum provides flexibility for teachers to address students' needs and interests. College teachers have an opportunity to develop courses for implementation across ACT schools. Based on the courses that have been accredited by the BSSS, college teachers are responsible for developing programs of learning. A program of learning is developed by individual colleges to implement the courses and units they are delivering.

Teachers must deliver all content descriptions; however, they do have flexibility to emphasise some content descriptions over others. It is at the discretion of the teacher to select the texts or materials to demonstrate the content descriptions. Teachers can choose to deliver course units in any order and teach additional (not listed) content provided it meets the specific unit goals.

School-based continuous assessment means that students are continually assessed throughout years 11 and 12, with both years contributing equally to senior secondary certification. Teachers and students are positioned to have ownership of senior secondary assessment. The system allows teachers to learn from each other and to refine their judgement and develop expertise.

Senior secondary teachers have the flexibility to assess students in a variety of ways. For example: multimedia presentation, inquiry-based project, test, essay, performance and/or practical demonstration may all have their place. College teachers are responsible for developing assessment instruments with task specific rubrics and providing feedback to students.

The integrity of the ACT Senior Secondary Certificate is upheld by a robust, collaborative and rigorous structured consensus-based peer reviewed moderation process. System moderation involves all Year 11 and 12 teachers from public, non-government and international colleges delivering the ACT Senior Secondary Certificate.

Only students who desire a pathway to university are required to sit a general aptitude test, referred to as the ACT Scaling Test (AST), which moderates student course scores across subjects and colleges. Students are required to use critical and creative thinking skills across a range of disciplines to solve problems. They are also required to interpret a stimulus and write an extended response.

Senior secondary curriculum makes provision for student-centred teaching approaches, integrated and project-based learning inquiry, formative assessment and teacher autonomy. ACT Senior Secondary Curriculum makes provision for diverse learners and students with mild to moderate intellectual disabilities, so that all students can achieve an ACT Senior Secondary Certificate.

The ACT Board of Senior Secondary Studies (BSSS) leads senior secondary education. It is responsible for quality assurance in senior secondary curriculum, assessment and certification. The Board consists of representatives from colleges, universities, industry, parent organisations and unions. The Office of the Board of Senior Secondary Studies (OBSSS) consists of professional and administrative staff who support the Board in achieving its objectives and functions.

ACT Senior Secondary Certificate

Courses of study for the ACT Senior Secondary Certificate:

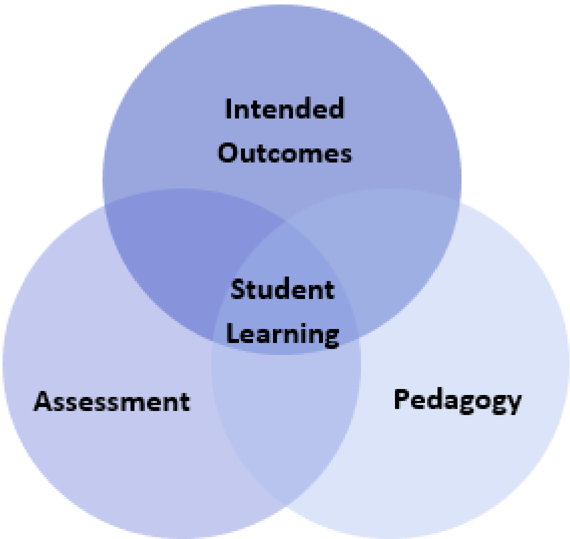
- provide a variety of pathways, to meet different learning needs and encourage students to complete their secondary education
- enable students to develop the essential capabilities for twenty-first century learners
- empower students as active participants in their own learning
- engage students in contemporary issues relevant to their lives
- foster students' intellectual, social and ethical development
- nurture students' wellbeing, and physical and spiritual development
- enable effective and respectful participation in a diverse society.

Each course of study:

- comprises an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum
- is based on a model of learning that integrates intended student outcomes, pedagogy and assessment
- outlines teaching strategies which are grounded in learning principles and encompass quality teaching
- promotes intellectual quality, establish a rich learning environment and generate relevant connections between learning and life experiences
- provides formal assessment and certification of students' achievements.

Underpinning beliefs

- All students are able to learn.
- Learning is a partnership between students and teachers.
- Teachers are responsible for advancing student learning.



Learning Principles

1. Learning builds on existing knowledge, understandings and skills.
   (Prior knowledge)
2. When learning is organised around major concepts, principles and significant real world issues, within and across disciplines, it helps students make connections and build knowledge structures.
   (Deep knowledge and connectedness)
3. Learning is facilitated when students actively monitor their own learning and consciously develop ways of organising and applying knowledge within and across contexts.
   (Metacognition)
4. Learners' sense of self and motivation to learn affects learning.
   (Self-concept)
5. Learning needs to take place in a context of high expectations.
   (High expectations)
6. Learners learn in different ways and at different rates.
   (Individual differences)
7. Different cultural environments, including the use of language, shape learners' understandings and the way they learn.
   (Socio-cultural effects)
8. Learning is a social and collaborative function as well as an individual one.
   (Collaborative learning)
9. Learning is strengthened when learning outcomes and criteria for judging learning are made explicit and when students receive frequent feedback on their progress.
   (Explicit expectations and feedback)

General Capabilities

All courses of study for the ACT Senior Secondary Certificate should enable students to develop essential capabilities for twenty-first century learners. These ‘capabilities’ comprise an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum.

The capabilities include:

- literacy
- numeracy
- information and communication technology (ICT)
- critical and creative thinking
- personal and social
- ethical behaviour
- intercultural understanding

Courses of study for the ACT Senior Secondary Certificate should be both relevant to the lives of students and incorporate the contemporary issues they face. Hence, courses address the following three priorities. These priorities are:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia’s engagement with Asia
- Sustainability

Elaboration of these General Capabilities and priorities is available on the ACARA website at www.australiancurriculum.edu.au.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains financial applications of Mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age. Students will also learn about the probability of certain events occurring and will therefore be well equipped to make informed decisions.

Information and Communication Technology (ICT) Capability in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and Creative Thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and Social Capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical Understanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural Understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Cross-Curriculum Priorities

Aboriginal and Torres Strait Islander Histories and Cultures

The senior secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

Asia and Australia's Engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss sustainability and learn the importance of respecting and valuing a wide range of world perspectives.

Mathematical Methods T

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real-world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

The major themes of Mathematical Methods are calculus and statistics. They include as necessary prerequisites studies of algebra, functions and their graphs, and probability. They are developed systematically, with increasing levels of sophistication and complexity. Calculus is essential for developing an understanding of the physical world because many of the laws of science are relationships involving rates of change. Statistics is used to describe and analyse phenomena involving uncertainty and variation. For these reasons this subject provides a foundation for further studies in disciplines in which mathematics and statistics have important roles. It is also advantageous for further studies in the health and social sciences. In summary, the subject Mathematical Methods is designed for students whose future pathways may involve mathematics and statistics and their applications in a range of disciplines at the tertiary level.

For all content areas of Mathematical Methods, the proficiency strands of the F-10 curriculum are still applicable and should be inherent in students' learning of this subject. These strands are Understanding, Fluency, Problem solving and Reasoning, and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as calculating derivatives and integrals, or solving quadratic equations, and frees up working memory for more complex aspects of problem solving. The ability to transfer skills to solve problems based on a wide range of applications is a vital part of mathematics in this subject. Because both calculus and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout this subject.

Mathematical Methods is structured over four units. The topics in Unit 1 build on students' mathematical experience. The topics 'Functions and graphs', 'Trigonometric functions' and 'Counting and probability' all follow on from topics in the F-10 curriculum from the strands, Number and Algebra, Measurement and Geometry and Statistics and Probability. In Mathematical Methods there is a progression of content and applications in all areas. For example, in Unit 2 differential calculus is introduced, and then further developed in Unit 3 where integral calculus is introduced. Discrete probability distributions are introduced in Unit 3, and then continuous probability distributions and an introduction to statistical inference conclude Unit 4.

Goals

Mathematical Methods aims to develop students':

- understanding of concepts and techniques drawn from algebra, the study of functions, calculus, probability and statistics
- ability to solve applied problems using concepts and techniques drawn from algebra, functions, calculus, probability and statistics
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Student Group

Links to Foundation to Year 10

In Mathematical Methods, there is a strong emphasis on mutually reinforcing proficiencies in Understanding, Fluency, Problem solving and Reasoning. Students gain fluency in a variety of mathematical and statistical skills, including algebraic manipulations, constructing and interpreting graphs, calculating derivatives and integrals, applying probabilistic models, estimating probabilities and parameters from data, and using appropriate technologies. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In order to study Mathematical Methods, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended for the study of Mathematical Methods:

- define rational and irrational numbers, and perform operations with surds and fractional indices
- factorise monic and non-monic quadratic expressions, and solve a wide range of quadratic equations derived from a variety of contexts
- calculate and interpret the mean and standard deviation of data, and use these to compare datasets.

Unit Titles

- Unit 1: Mathematical Methods
- Unit 2: Mathematical Methods
- Unit 3: Mathematical Methods
- Unit 4: Mathematical Methods
- Unit 5: Mathematical Methods

Organisation of Content

Mathematical Methods focuses on the development of the use of calculus and statistical analysis. The study of calculus in Mathematical Methods provides a basis for an understanding of the physical world involving rates of change, and includes the use of functions, their derivatives and integrals, in modelling physical processes. The study of statistics in Mathematical Methods develops the ability to describe and analyse phenomena involving uncertainty and variation.

Mathematical Methods is organised into four units. The topics broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. The probability and statistics topics lead to an introduction to statistical inference.

	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**
Mathematical Methods	• Functions and graphs • Trigonometric functions • Counting and probability	• Exponential functions • Arithmetic and geometric sequences and series • Introduction to differential calculus	• Further differentiation and applications • Integrals • Discrete random variables	• The logarithmic function • Continuous random variables and the normal distribution • Interval estimates for proportions

Unit 1: Mathematical Methods

Unit 1 begins with a review of the basic algebraic concepts and techniques required for a successful introduction to the study of functions and calculus. Simple relationships between variable quantities are reviewed, and these are used to introduce the key concepts of a function and its graph.
The study of probability and statistics begins in this unit with a review of the fundamentals of probability, and the introduction of the concepts of conditional probability and independence.
The study of the trigonometric functions begins with a consideration of the unit circle using degrees and the trigonometry of triangles and its application. Radian measure is introduced, and the graphs of the trigonometric functions are examined and their applications in a wide range of settings are explored.

Unit 2: Mathematical Methods

In **Unit 2**, exponential functions are introduced and their properties and graphs examined. Arithmetic and geometric sequences and their applications are introduced and their recursive definitions applied. Rates and average rates of change are introduced, and this is followed by the key concept of the derivative as an 'instantaneous rate of change'. These concepts are reinforced numerically (by calculating difference quotients), geometrically (as slopes of chords and tangents), and algebraically. This first calculus topic concludes with derivatives of polynomial functions, using simple applications of the derivative to sketch curves, calculate slopes and equations of tangents, determine instantaneous velocities, and solve optimisation problems.

Unit 3: Mathematical Methods

In **Unit 3**, the study of calculus continues by introducing the derivatives of exponential and trigonometric functions and their applications, as well as some basic differentiation techniques and the concept of a second derivative, its meaning and applications. The aim is to demonstrate to students the beauty and power of calculus and the breadth of its applications. The unit includes integration, both as a process that reverses differentiation and as a way of calculating areas. The fundamental theorem of calculus as a link between differentiation and integration is emphasised. Discrete random variables are introduced, together with their uses in modelling random processes involving chance and variation. The purpose here is to develop a framework for statistical inference.

Unit 4: Mathematical Methods

In **Unit 4**, the logarithmic function and its derivative are studied. Continuous random variables are introduced and their applications examined. Probabilities associated with continuous distributions are calculated using definite integrals. In this unit students are introduced to one of the most important parts of statistics, namely statistical inference, where the goal is to estimate an unknown parameter associated with a population using a sample of that population. In this unit, inference is restricted to estimating proportions in two-outcome populations. Students will already be familiar with many examples of these types of populations.

Unit 5: Mathematical Methods

Unit 5 combines Unit 3b and Unit 4a.

Assessment

The identification of criteria within the achievement standards and assessment task types and weightings provides a common and agreed basis for the collection of evidence of student achievement.

Assessment Criteria (the dimensions of quality that teachers look for in evaluating student work) provide a common and agreed basis for judgement of performance against unit and course goals, within and across colleges. Over a course, teachers must use all these criteria to assess students' performance but are not required to use all criteria on each task. Assessment criteria are to be used holistically on a given task and in determining the unit grade.

Assessment Tasks elicit responses that demonstrate the degree to which students have achieved the goals of a unit based on the assessment criteria. The Common Curriculum Elements (CCE) is a guide to developing assessment tasks that promote a range of thinking skills (see Appendix C). It is highly desirable that assessment tasks engage students in demonstrating higher order thinking.

Rubrics are constructed for individual tasks, informing the assessment criteria relevant for a particular task and can be used to assess a continuum that indicates levels of student performance against each criterion.

Assessment Criteria

Students will be assessed on the degree to which they demonstrate an understanding of:

- concepts and techniques
- reasoning and communications.

Assessment Task Types

Suggested tasks:

- project/assignment
- modelling projects
- portfolio
- journal
- validation activity
- presentation such as a pitch, poster, vodcast, interview
- practical activity such as a demonstration
- test/examination
- online adaptive tasks/quiz

Weightings in T 1.0 Units:

No task to be weighted more than 50% for a standard 1.0 unit.

Additional Assessment Information

Requirements

- For a standard unit (1.0), students must complete a minimum of three assessment tasks and a maximum of five.
- For a half standard unit (0.5), students must complete a minimum of two and a maximum of three assessment tasks.
- Students should experience a variety of task types (test and non-test) and different modes of communication to demonstrate the Achievement Standards.
- Students are required to undertake at least one problem solving investigation task each semester. This task may be completed individually or collaboratively. They are required to plan, enquire into and draw conclusions about key unit concepts. Students may respond in forms such as modelling projects, problem solving and practical activities.
- Assessment tasks for a standard (1.0) or half-standard (0.5) unit must be informed by the Achievement Standards.

Advice

- It is recommended that the total component of unsupervised tasks be no greater than 30%.
- For tasks completed in unsupervised conditions, schools need to have mechanisms to uphold academic integrity, for example, student declaration, plagiarism software, oral defence, interview, other validation tasks

Achievement Standards

Years 11 and 12 achievement standards are written for A/T courses. A single achievement standard is written for M courses.

A Year 12 student in any unit is assessed using the Year 12 achievement standards. A Year 11 student in any unit is assessed using the Year 11 achievement standards. Year 12 achievement standards reflect higher expectations of student achievement compared to the Year 11 achievement standards. Years 11 and 12 achievement standards are differentiated by cognitive demand, the number of dimensions and the depth of inquiry.

An achievement standard cannot be used as a rubric for an individual assessment task. Assessment is the responsibility of the college. Student tasks may be assessed using rubrics or marking schemes devised by the college. A teacher may use the achievement standards to inform development of rubrics. The verbs used in achievement standards may be reflected in the rubric. In the context of combined Years 11 and 12 classes, it is best practice to have a distinct rubric for Years 11 and 12. These rubrics should be available for students prior to completion of an assessment task so that success criteria are clear.

Student achievement in A, T and M units is reported based on system standards as an A-E grade. Grade descriptors and standard work samples where available, provide a guide for teacher judgement of students' achievement over the unit.

Grades are awarded on the proviso that the assessment requirements have been met. Teachers will consider, when allocating grades, the degree to which students demonstrate their ability to complete and submit tasks within a specified time frame.

Achievement Standards for Mathematics T Course – Year 11

	*A student who achieves an **A** grade typically*	*A student who achieves a **B** grade typically*	*A student who achieves a **C** grade typically*	*A student who achieves a **D** grade typically*	*A student who achieves an **E** grade typically*
Concepts and Techniques	• critically applies mathematical concepts in a variety of complex contexts to routine and non-routine problems • selects and applies advanced mathematical techniques to solve complex problems in a variety of contexts • constructs, selects and applies complex mathematical models to routine and non-routine problems in a variety of contexts • uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts	• applies mathematical concepts in a variety of contexts to routine and non-routine problems • selects and applies mathematical techniques to solve routine and non-routine problems in a variety of contexts • selects and applies mathematical models to routine and non-routine problems to a variety of contexts • uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts	• applies mathematical concepts in some contexts to routine and non-routine problems • applies mathematical techniques to solve routine and non-routine problems in some contexts • applies mathematical models to routine and non-routine problems in some contexts • uses digital technologies appropriately to solve routine and non-routine problems in some contexts	• applies simple mathematical concepts in limited contexts to routine problems • applies simple mathematical techniques to solve routine problems in limited contexts • applies simple mathematical models to routine problems in limited contexts • uses digital technologies appropriately to solve routine problems in limited contexts	• applies simple mathematical concepts in structured contexts • uses simple mathematical techniques to solve routine problems in structured contexts • demonstrates limited familiarity with mathematical models in structured contexts • uses digital technologies to solve routine problems in structured contexts
Reasoning and Communications	• represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and well-reasoned, using appropriate and accurate language • evaluates the reasonableness of solutions to routine and non-routine problems in a variety of contexts • reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively • evaluates the potential of Mathematics to generate knowledge in the public good	• represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems a variety of contexts • communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language • analyses the reasonableness of solutions to routine and non-routine problems • reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively • analyses the potential of Mathematics to generate knowledge in the public good	• represents mathematical concepts in numerical, graphical and symbolic form to some routine and some non-routine problems in some contexts • communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language • explains the reasonableness of solutions to some routine and non-routine problems • reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively • explains the potential of Mathematics to generate knowledge in the public good	• represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in limited contexts • communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language • describes the appropriateness of solutions to routine problems • reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively • describes the potential of Mathematics to generate knowledge in the public good	• represents simple mathematical concepts in numerical, graphical or symbolic form in structured contexts • communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language • identifies solutions to routine problems in structured contexts • reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively • identifies some ways in which Mathematics is used to generate knowledge in the public good

Achievement Standards for Mathematics T Course – Year 12

	*A student who achieves an **A** grade typically*	*A student who achieves a **B** grade typically*	*A student who achieves a **C** grade typically*	*A student who achieves a **D** grade typically*	*A student who achieves an **E** grade typically*
Concepts and Techniques	• critically and creatively applies mathematical concepts in a variety of complex contexts to routine and non-routine problems • synthesises information to select and apply mathematical techniques to solve complex problems in a variety of contexts • constructs, selects and applies mathematical models to a variety of contexts in routine and non-routine problems • uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts	• critically applies mathematical concepts in a variety of contexts to routine and non-routine problems • analyses information to select and apply mathematical techniques to solve routine and non-routine problems in a variety of contexts • selects and applies mathematical models to routine and non-routine problems in a variety of contexts • uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts	• applies mathematical concepts in some contexts to routine and non-routine problems • selects and applies mathematical techniques to solve routine and some non-routine problems in some contexts • applies mathematical models to routine and non-routine problems in some contexts • uses digital technologies appropriately to solve routine and non-routine problems in a variety of contexts	• applies simple mathematical concepts in limited contexts to routine problems • applies simple mathematical techniques to solve routine problems in limited contexts • applies simple mathematical models to routine problems in limited contexts • uses digital technologies appropriately to solve routine problems in limited contexts	• applies simple mathematical concepts in structured contexts • uses simple mathematical techniques to solve routine problems in structured contexts • demonstrates limited familiarity with mathematical models to solve routine problems in structured contexts • uses digital technologies to solve routine problems in structured contexts
Reasoning and Communications	• represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and reasoned, using appropriate and accurate language • evaluates the solutions to routine and non-routine problems in a variety of contexts • evaluates methods and models for their strengths and limitations when developing solutions to routine and non-routine problems • reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively • evaluates the potential of Mathematics to generate knowledge in the public good	• represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts • communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language • analyses the solutions to routine and non-routine problems in some contexts • analyses strengths and limitations of models used when developing solutions to routine and non-routine problems • reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively • analyses the potential of Mathematics to generate knowledge in the public good	• represents mathematical concepts in numerical, graphical and symbolic form in some routine and non-routine problems in some contexts • communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language • explains solutions to some routine and non-routine problems in some contexts • explains strengths and limitations of models used when developing solutions to some routine and non-routine problems • reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively • explains the potential of Mathematics to generate knowledge in the public good	• represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in structured contexts • communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language • describes solutions to routine problems in limited contexts • describes strengths or limitations of simple models when solving routine problems • reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively • describes the potential of Mathematics to generate knowledge in the public good	• represents simple mathematical concepts in numerical, graphical or symbolic form in in simple problems in structured contexts • communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language • identifies solutions to routine problems in structured contexts • identifies strengths or limitations of simple models in relation to routine problems • reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively • identifies some ways in which Mathematics is used to generate knowledge in the public good

Unit 1: Mathematical Methods — Value: 1.0

Unit 1a: Mathematical Methods — **Value: 0.5**

Unit 1b: Mathematical Methods — **Value: 0.5**

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in algebra, functions, graphs, trigonometric functions and probability
- solve problems using algebra, functions, graphs, trigonometric functions and probability
- apply reasoning skills in the context of algebra, functions, graphs, trigonometric functions and probability
- interpret and evaluate mathematical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Functions and graphs

Lines and linear relationships

- determine the coordinates of the midpoint of two points
- examine examples of direct proportion and linearly related variables
- recognise features of the graph of $y = mx + c$, including its linear nature, its intercepts and its slope or gradient
- find the equation of a straight line given sufficient information; parallel and perpendicular lines
- solve linear equations.

Review of quadratic relationships:

- examine examples of quadratically related
- recognise features of the graphs of $y = x^2$, $y = a(x - b)^2 + c$, and $y = a(x - b)(x - c)$, including their parabolic nature, turning points, axes of symmetry and intercepts
- solve quadratic equations using the quadratic formula and by completing the square
- find the equation of a quadratic given sufficient information
- find turning points and zeros of quadratics and understand the role of the discriminant
- recognise features of the graph of the general quadratic $y = ax^2 + bx + c$.

Inverse proportion:

- examine examples of inverse proportion
- recognise features of the graphs of $y = \frac{1}{x}$ and $y = \frac{a}{x-b}$, including their hyperbolic shapes, and their asymptotes.

Powers and polynomials:

- recognise features of the graphs of $y = x^n$ for $n \in \boldsymbol{N}$, $n = -1$ and $n =$ ½, including shape, and behaviour as $x \to \infty$ and $x \to -\infty$
- identify the coefficients and the degree of a polynomial
- expand quadratic and cubic polynomials from factors
- recognise features of the graphs of $y = x^3$, $y = a(x-b)^3 + c$ and $y = k(x-a)(x-b)(x-c)$, including shape, intercepts and behaviour as $x \to \infty$ and $x \to -\infty$
- factorise cubic polynomials in cases where a linear factor is easily obtained
- solve cubic equations using technology, and algebraically in cases where a linear factor is easily obtained.

Graphs of relations:

- recognise features of the graphs of $x^2 + y^2 = r^2$ and $(x-a)^2 + (y-b)^2 = r^2$, including their circular shapes, their centres and their radii
- recognise features of the graph of $y^2 = x$ including its parabolic shape and its axis of symmetry.

Functions:

- understand the concept of a function as a mapping between sets, and as a rule or a formula that defines one variable quantity in terms of another
- use function notation, domain and range, independent and dependent variables
- understand the concept of the graph of a function
- examine translations and the graphs of $y = f(x) + a$ and $y = f(x+b)$
- examine dilations and the graphs of $y = cf(x)$ and $y = f(kx)$
- recognise the distinction between functions and relations, and the vertical line test.

Topic 2: Trigonometric functions

Cosine and sine rules:

- review sine, cosine and tangent as ratios of side lengths in right-angled triangles
- understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using degrees
- examine the relationship between the angle of inclination of a line and the gradient of that line
- establish and use the sine and cosine rules and the formula $Area = \frac{1}{2}bc\sin A$ for the area of a triangle.

Circular measure and radian measure:

- define and use radian measure and understand its relationship with degree measure
- calculate lengths of arcs and areas of sectors in circles.

Trigonometric functions:

- understand the unit circle definition of $\cos\theta$, $\sin\theta$ and $\tan\theta$ and periodicity using radians
- recognise the exact values of $\sin\theta$, $\cos\theta$ and $\tan\theta$ at integer multiples of $\frac{\pi}{6}$ and $\frac{\pi}{4}$
- recognise the graphs of $y = \sin x, y = \cos x$, and $y = \tan x$ on extended domains
- examine amplitude changes and the graphs of $y = a\sin x$ and $y = a\cos x$
- examine period changes and the graphs of $y = \sin bx$, $y = \cos bx$, and $y = \tan\ bx$
- examine phase changes and the graphs of $y = \sin(x + c)$, $y = \cos(x + c)$ and $y = \tan\ (x + c)$ and the relationships $\sin\left(x + \frac{\pi}{2}\right) = \cos x$ and $\cos\left(x - \frac{\pi}{2}\right) = \sin x$
- prove and apply the angle sum and difference identities
- identify contexts suitable for modelling by trigonometric functions and use them to solve practical problems
- solve equations involving trigonometric functions using technology, and algebraically in simple cases.

Topic 3: Counting and Probability

Combinations:

- understand the notion of a combination as an unordered set of r objects taken from a set of n distinct objects
- use the notation $\binom{n}{r}$ and the formula $\binom{n}{r} = \frac{n!}{r!(n-r)!}$ for the number of combinations of r objects taken from a set of n distinct objects
- expand $(x + y)^n$ for small positive integers n
- recognise the numbers $\binom{n}{r}$ as binomial coefficients, (as coefficients in the expansion of $(x + y)^n$)
- use Pascal's triangle and its properties.

Language of events and sets:

- review the concepts and language of outcomes, sample spaces and events as sets of outcomes
- use set language and notation for events, including $\overline{A}$ (or A') for the complement of an event A, $A \cap B$ for the intersection of events A and B, and $A \cup B$ for the union, and recognise mutually exclusive events
- use everyday occurrences to illustrate set descriptions and representations of events, and set operations.

Review of the fundamentals of probability:

- review probability as a measure of 'the likelihood of occurrence' of an event review the probability scale: $0 \leq P(A) \leq 1$ for each event A, with $P(A) = 0$ if A is an impossibility and $P(A) = 1$ if A is a certainty
- review the rules: $P(A') = 1 - P(A)$ and $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
- use relative frequencies obtained from data as point estimates of probabilities.

Conditional probability and independence:

- understand the notion of a conditional probability and recognise and use language that indicates conditionality
- use the notation $P(A|B)$ and the formula $P(A \cap B) = P(A|B)P(B)$

- understand the notion of independence of an event A from an event B, as defined by $P(A|B) = P(A)$
- establish and use the formula $P(A \cap B) = P(A)P(B)$ for independent events A and B, and recognise the symmetry of independence
- use relative frequencies obtained from data as point estimates of conditional probabilities and as indications of possible independence of events.

A guide to reading and implementing content descriptions

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Assessment

Refer to pages 11-13.

Unit 2: Mathematical Methods — Value: 1.0

Unit 2a: Mathematical Methods — Value: 0.5

Unit 2b: Mathematical Methods — Value: 0.5

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques used in algebra, sequences and series, functions, graphs and calculus
- solve problems in algebra, sequences and series, functions, graphs and calculus
- apply reasoning skills in algebra, sequences and series, functions, graphs and calculus
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate arguments and strategies when solving problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Exponential functions

Indices and the index laws:

- review indices (including fractional indices) and the index laws
- use radicals and convert to and from fractional indices understand and use scientific notation and significant figures.

Exponential functions:

- establish and use the algebraic properties of exponential functions
- recognise the qualitative features of the graph of $y = a^x$ $(a > 0)$ including asymptotes, and of its translations ($y = a^x + b$ and $y = a^{x+c}$)
- identify contexts suitable for modelling by exponential functions and use them to solve practical problems
- solve equations involving exponential functions using technology, and algebraically in simple cases.

Topic 2 Arithmetic and geometric sequences and series

Arithmetic sequences:

- recognise and use the recursive definition of an arithmetic sequence: $t_{n+1} = t_n + d$
- use the formula $t_n = t_1 + (n-1)d$ for the general term of an arithmetic sequence and recognise its linear nature
- use arithmetic sequences in contexts involving discrete linear growth or decay, such as simple interest
- establish and use the formula for the sum of the first n terms of an arithmetic sequence.

Geometric sequences:

- recognise and use the recursive definition of a geometric sequence: $t_{n+1} = rt_n$
- use the formula $t_n = r^{n-1}t_1$ for the general term of a geometric sequence and recognise its exponential nature
- understand the limiting behaviour as $n \to \infty$ of the terms t_n in a geometric sequence and its dependence on the value of the common ratio r
- establish and use the formula $S_n = t_1 \frac{r^n - 1}{r - 1}$ for the sum of the first n terms of a geometric sequence
- use geometric sequences in contexts involving geometric growth or decay, such as compound interest.

Topic 3: Introduction to differential calculus

Rates of change:

- interpret the difference quotient $\frac{f(x+h)-f(x)}{h}$ as the average rate of change of a function f
- use the Leibniz notation δx and δy for changes or increments in the variables x and y
- use the notation $\frac{\delta y}{\delta x}$ for the difference quotient $\frac{f(x+h)-f(x)}{h}$ where $y = f(x)$
- interpret the ratios $\frac{f(x+h)-f(x)}{h}$ and $\frac{\delta y}{\delta x}$ as the slope or gradient of a chord or secant of the graph of $y = f(x)$.

The concept of the derivative:

- examine the behaviour of the difference quotient $\frac{f(x+h)-f(x)}{h}$ as $h \to 0$ as an informal introduction to the concept of a limit
- define the derivative $f'(x)$ as $\lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$
- use the Leibniz notation for the derivative: $\frac{dy}{dx} = \lim_{\delta x \to 0} \frac{\delta y}{\delta x}$ and the correspondence $\frac{dy}{dx} = f'(x)$ where $y = f(x)$
- interpret the derivative as the instantaneous rate of change
- interpret the derivative as the slope or gradient of a tangent line of the graph of $y = f(x)$.

Computation of derivatives:

- estimate numerically the value of a derivative, for simple power functions
- examine examples of variable rates of change of non-linear functions
- establish the formula $\frac{d}{dx}(x^n) = nx^{n-1}$ for positive integers n by expanding $(x+h)^n$ or by factorising $(x+h)^n - x^n$.

Properties of derivatives:

- understand the concept of the derivative as a function
- recognise and use linearity properties of the derivative
- calculate derivatives of polynomials and other linear combinations of power functions.

Applications of derivatives:

- find instantaneous rates of change
- find the slope of a tangent and the equation of the tangent
- construct and interpret position-time graphs, with velocity as the slope of the tangent
- sketch curves associated with simple polynomials; find stationary points, and local and global maxima and minima; and examine behaviour as $x \to \infty$ and $x \to -\infty$
- solve optimisation problems arising in a variety of contexts involving simple polynomials on finite interval domains.

Anti-derivatives:

- calculate anti-derivatives of polynomial functions and apply to solving simple problems involving motion in a straight line.

A guide to reading and implementing content descriptions

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Assessment

Refer to pages 11-13.

Unit 3: Mathematical Methods Value: 1.0

Unit 3a: Mathematical Methods Value: 0.5

Unit 3b: Mathematical Methods Value: 0.5

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems
- communicate their arguments and strategies when solving problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Further differentiation and applications

Exponential functions:

- estimate the limit of $\frac{a^h-1}{h}$ as $h \to 0$ using technology, for various values of $a > 0$
- recognise that e is the unique number a for which the above limit is 1
- establish and use the formula $\frac{d}{dx}(e^x) = e^x$
- use exponential functions and their derivatives to solve practical problems. Trigonometric functions:
- establish the formulas $\frac{d}{dx}(\sin x) = \cos x$, and $\frac{d}{dx}(\cos x) = -\sin x$ by numerical estimations of the limits and informal proofs based on geometric constructions
- use trigonometric functions and their derivatives to solve practical problems.

Differentiation rules:

- understand and use the product and quotient rules
- understand the notion of composition of functions and use the chain rule for determining the derivatives of composite functions
- apply the product, quotient and chain rule to differentiate functions such as xe^x, $\tan x$, $\frac{1}{x^n}$, $x \sin x$, $e^{-x} \sin x$ and $f(ax + b)$.

The second derivative and applications of differentiation:

- use the increments formula: $\delta y \cong \frac{dy}{dx} \times \delta x$ to estimate the change in the dependent variable y resulting from changes in the independent variable x
- understand the concept of the second derivative as the rate of change of the first derivative function
- recognise acceleration as the second derivative of position with respect to time
- understand the concepts of concavity and points of inflection and their relationship with the second derivative
- understand and use the second derivative test for finding local maxima and minima
- sketch the graph of a function using first and second derivatives to locate stationary points and points of inflection
- solve optimisation problems from a wide variety of fields using first and second derivatives.

Topic 2: Integrals

Anti-differentiation:

- recognise anti-differentiation as the reverse of differentiation
- use the notation $\int f(x)dx$ for anti-derivatives or indefinite integrals
- establish and use the formula $\int x^n dx = \frac{1}{n+1}x^{n+1} + c$ for $n \neq -1$
- establish and use the formula $\int e^x dx = e^x + c$
- establish and use the formulas $\int \sin x \, dx = -\cos x + c$ and $\int \cos x \, dx = \sin x + c$
- recognise and use linearity of anti-differentiation
- determine indefinite integrals of the form $\int f(ax + b)dx$
- identify families of curves with the same derivative function
- determine $f(x)$, given $f'(x)$and an initial condition $f(a) = b$
- determine displacement given velocity in linear motion problems.

Definite integrals:

- examine the area problem, and use sums of the form $\sum_i f(x_i)\, \delta x_i$ to estimate the area under the curve $y = f(x)$
- interpret the definite integral $\int_a^b f(x)dx$ as area under the curve $y = f(x)$ if $f(x) > 0$
- recognise the definite integral $\int_a^b f(x)dx$ as a limit of sums of the form $\sum_i f(x_i)\, \delta x_i$
- interpret $\int_a^b f(x)dx$ as a sum of signed areas (
- recognise and use the additivity and linearity of definite integrals.

Fundamental theorem:

- understand the concept of the signed area function $F(x) = \int_a^x f(t)dt$
- understand and use the theorem: $F'(x) = \frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$, and illustrate its proof geometrically
- understand the formula $\int_a^b f(x)dx = F(b) - F(a)$ and use it to calculate definite integrals.

Applications of integration:

- calculate the area under a curve
- calculate total change by integrating instantaneous or marginal rate of change
- calculate the area between curves in simple cases
- determine positions given acceleration and initial values of position and velocity.

Topic 3: Discrete random variables

General discrete random variables:

- understand the concepts of a discrete random variable and its associated probability function, and their use in modelling data
- use relative frequencies obtained from data to obtain point estimates of probabilities associated with a discrete random variable
- recognise uniform discrete random variables and use them to model random phenomena with equally likely outcomes
- examine simple examples of non-uniform discrete random variables
- recognise the mean or expected value of a discrete random variable as a measurement of centre, and evaluate it in simple cases
- recognise the variance and standard deviation of a discrete random variable as measures of spread, and evaluate them in simple cases
- use discrete random variables and associated probabilities to solve practical problems.

Bernoulli distributions:

- use a Bernoulli random variable as a model for two-outcome situations
- identify contexts suitable for modelling by Bernoulli random variables
- recognise the mean p and variance $p(1-p)$ of the Bernoulli distribution with parameter p
- use Bernoulli random variables and associated probabilities to model data and solve practical problems.

Binomial distributions:

- understand the concepts of Bernoulli trials and the concept of a binomial random variable as the number of 'successes' in n independent Bernoulli trials, with the same probability of success p in each trial
- identify contexts suitable for modelling by binomial random variables
- determine and use the probabilities $P(X=r)=\binom{n}{r}p^r(1-p)^{n-r}$ associated with the binomial distribution with parameters n and p; note the mean np and variance $np(1-p)$ of a binomial distribution
- use binomial distributions and associated probabilities to solve practical problems.

A guide to reading and implementing content descriptions

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A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 11-13.

Unit 4: Mathematical Methods — Value: 1.0

Unit 4a: Mathematical Methods — **Value: 0.5**

Unit 4b: Mathematical Methods — **Value: 0.5**

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in calculus, probability and statistics
- solve problems in calculus, probability and statistics
- apply reasoning skills in calculus, probability and statistics
- interpret and evaluate mathematical and statistical information and ascertain the reasonableness of solutions to problems.
- communicate their arguments and strategies when solving problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: The Logarithmic function

Logarithmic functions:

- define logarithms as indices: $a^x = b$ is equivalent to $x = \log_a b$ i.e. $a^{\log_a b} = b$
- establish and use the algebraic properties of logarithms
- recognise the inverse relationship between logarithms and exponentials: $y = a^x$ is equivalent to $x = \log_a y$
- interpret and use logarithmic scales such as decibels in acoustics, the Richter Scale for earthquake magnitude, octaves in music, pH in chemistry
- solve equations involving indices using logarithms
- recognise the qualitative features of the graph of $y = \log_a x$ $(a > 1)$ including asymptotes, and of its translations $y = \log_a x + b$ and $y = \log_a(x + c)$ (
- solve simple equations involving logarithmic functions algebraically and
- identify contexts suitable for modelling by logarithmic functions and use them to solve practical problems. Calculus of logarithmic functions:
- define the natural logarithm $\ln x = \log_e x$
- recognise and use the inverse relationship of the functions $y = e^x$ and $y = \ln x$
- establish and use the formula $\frac{d}{dx}(\ln x) = \frac{1}{x}$ (
- establish and use the formula $\int \frac{1}{x} dx = \ln x + c$, for $x > 0$
- use logarithmic functions and their derivatives to solve practical problems.

Topic 2: Continuous random variables and the normal distribution

General continuous random variables:

- use relative frequencies and histograms obtained from data to estimate probabilities associated with a continuous random
- understand the concepts of a probability density function, cumulative distribution function, and probabilities associated with a continuous random variable given by integrals; examine simple types of continuous random variables and use them in appropriate contexts
- recognise the expected value, variance and standard deviation of a continuous random variable and evaluate them in simple cases
- understand the effects of linear changes of scale and origin on the mean and the standard deviation.

Normal distributions:

- identify contexts such as naturally occurring variation that are suitable for modelling by normal random variables
- recognise features of the graph of the probability density function of the normal distribution with mean μ and standard deviation σ and the use of the standard normal distribution
- calculate probabilities and quantiles associated with a given normal distribution using technology, and use these to solve practical problems.

Topic 3: Interval estimates for proportions

Random sampling:

- understand the concept of a random sample
- discuss sources of bias in samples, and procedures to ensure randomness
- use graphical displays of simulated data to investigate the variability of random samples from various types of distributions, including uniform, normal and Bernoulli.

Sample proportions:

- understand the concept of the sample proportion $\hat{p}$ as a random variable whose value varies between samples, and the formulas for the mean p and standard deviation $\sqrt{(p(1-p)/n}$ of the sample proportion $\hat{p}$
- examine the approximate normality of the distribution of $\hat{p}$ for large samples
- simulate repeated random sampling, for a variety of values of *p* and a range of sample sizes, to illustrate the distribution of $\hat{p}$ and the approximate standard normality of $\frac{\hat{p}-p}{\sqrt{(\hat{p}(1-\hat{p})/n}}$ where the closeness of the approximation depends on both *n* and *p.*

Confidence intervals for proportions:

- the concept of an interval estimate for a parameter associated with a random variable
- use the approximate confidence interval $\left(\hat{p}-z\sqrt{(\hat{p}(1-\hat{p})/n},\ \hat{p}+z\sqrt{(\hat{p}(1-\hat{p})/n}\right)$, as an interval estimate for p, where z is the appropriate quantile for the standard normal distribution
- define the approximate margin of error $E=z\sqrt{(\hat{p}(1-\hat{p})/n}$ and understand the trade-off between margin of error and level of confidence
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain p.

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A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 11-13.

Unit 5: Mathematical Methods Value: 1.0

(This unit combines Unit 3b and Unit 4a)

Appendix A – Implementation Guidelines

Available course patterns

A standard 1.0 value unit is delivered over at least 55 hours. To be awarded a course, students must complete at least the minimum units over the whole minor, major, major/minor or double major course.

Course	Number of standard units to meet course requirements
Minor	Minimum of 2 units
Major	Minimum of 3.5 units

Units in this course can be delivered in any order.

Prerequisites for the course or units within the course

Nil.

Arrangements for students continuing study in this course

Students who studied the previous course may undertake any units in this course provided there is no duplication of content.

Duplication of Content Rules

Students cannot be given credit towards the requirements for a Senior Secondary Certificate for a unit that significantly duplicates content in a unit studied in another course. The responsibility for preventing undesirable overlap of content studied by a student rests with the principal and the teacher delivering the course. Students will only be given credit for covering the content once.

Guidelines for Delivery

Program of Learning

A program of learning is what a school provides to implement the course for a subject. This meets the requirements for context, scope and sequence set out in the Board endorsed course. Students follow programs of learning in a college as part of their senior secondary studies. The detail, design and layout of a program of learning are a college decision.

The program of learning must be documented to show the planned learning activities and experiences that meet the needs of particular groups of students, taking into account their interests, prior knowledge, abilities and backgrounds. The program of learning is a record of the learning experiences that enable students to achieve the knowledge, understanding and skills of the content descriptions. There is no requirement to submit a program of learning to the OBSSS for approval. The Principal will need to sign off at the end of Year 12 that courses have been delivered as accredited.

Content Descriptions

Are all content descriptions of equal importance? No. It depends on the focus of study. Teachers can customise their program of learning to meet their own students' needs, adding additional content descriptions if desired or emphasising some over others. A teacher must balance student needs with their responsibility to teach all content descriptions. It is mandatory that teachers address all content descriptions and that students engage with all content descriptions.

Half standard 0.5 units

Half standard units appear on the course adoption form but are not explicitly documented in courses. It is at the discretion of the college principal to split a standard 1.0 unit into two half standard 0.5 units. Colleges are required to adopt the half standard 0.5 units. However, colleges are not required to submit explicit documentation outlining their half standard 0.5 units to the BSSS. Colleges must assess students using the half standard 0.5 assessment task weightings outlined in the framework. It is the responsibility of the college principal to ensure that all content is delivered in units approved by the Board.

Moderation

Moderation is a system designed and implemented to:

- provide comparability in the system of school-based assessment
- form the basis for valid and reliable assessment in senior secondary schools
- involve the ACT Board of Senior Secondary Studies and colleges in cooperation and partnership
- maintain the quality of school-based assessment and the credibility, validity and acceptability of Board certificates.

Moderation commences within individual colleges. Teachers develop assessment programs and instruments, apply assessment criteria, and allocate Unit Grades, according to the relevant Course Framework. Teachers within course teaching groups conduct consensus discussions to moderate marking or grading of individual assessment instruments and unit grade decisions.

The Moderation Model

Moderation within the ACT encompasses structured, consensus-based peer review of Unit Grades for all accredited courses over two Moderation Days. In addition to Moderation Days, there is statistical moderation of course scores, including small group procedures, for T courses.

Moderation by Structured, Consensus-based Peer Review

Consensus-based peer review involves the review of student work against system wide criteria and standards and the validation of Unit Grades. This is done by matching student performance with the criteria and standards outlined in the Achievement Standards, as stated in the Framework. Advice is then given to colleges to assist teachers with, or confirm, their judgments. In addition, feedback is given on the construction of assessment instruments.

Preparation for Structured, Consensus-based Peer Review

Each year, teachers of Year 11 are asked to retain originals or copies of student work completed in Semester 2. Similarly, teachers of a Year 12 class should retain originals or copies of student work completed in Semester 1. Assessment and other documentation required by the Office of the Board of Senior Secondary Studies should also be kept. Year 11 work from Semester 2 of the previous year is presented for review at Moderation Day 1 in March, and Year 12 work from Semester 1 is presented for review at Moderation Day 2 in August.

In the lead up to Moderation Day, a College Course Presentation (comprised of a document folder and a set of student portfolios) is prepared for each A, T and M course/units offered by the school and is sent into the Office of the Board of Senior Secondary Studies.

The College Course Presentation

The package of materials (College Course Presentation) presented by a college for review on Moderation Days in each course area will comprise the following:

- a folder containing supporting documentation as requested by the Office of the Board through memoranda to colleges, including marking schemes and rubrics for each assessment item
- a set of student portfolios containing marked and/or graded written and non-written assessment responses and completed criteria and standards feedback forms. Evidence of all assessment responses on which the Unit Grade decision has been made is to be included in the student review portfolios.

Specific requirements for subject areas and types of evidence to be presented for each Moderation Day will be outlined by the Board Secretariat through the *Requirements for Moderation Memoranda* and Information Papers.

Visual evidence for judgements made about practical performances

It is a requirement that schools' judgements of standards to practical performances (A/T/M) be supported by visual evidence (still photos or video).

The photographic evidence submitted must be drawn from practical skills performed as part of the assessment process.

Teachers should consult the BSSS website for current information regarding all moderation requirements including subject specific and photographic evidence.

Appendix B – Course Developers

Name	College
Jacob Woolley	Canberra College
Gary Pocock	Canberra Institute of Technology
Marion McIntosh	Melba Copland Secondary School
Wayne Semmens	Melba Copland Secondary School
Jennifer Missen	Merici College
Nicole Burg	Narrabundah College
Rebecca Guinane	Narrabundah College
Andrew Trost	Narrabundah College

Appendix C – Common Curriculum Elements

Common curriculum elements assist in the development of high-quality assessment tasks by encouraging breadth and depth and discrimination in levels of achievement.

Organisers	Elements	Examples
create, compose and apply	apply	ideas and procedures in unfamiliar situations, content and processes in non-routine settings
	compose	oral, written and multimodal texts, music, visual images, responses to complex topics, new outcomes
	represent	images, symbols or signs
	create	creative thinking to identify areas for change, growth and innovation, recognise opportunities, experiment to achieve innovative solutions, construct objects, imagine alternatives
	manipulate	images, text, data, points of view
analyse, synthesise and evaluate	justify	arguments, points of view, phenomena, choices
	hypothesise	statement/theory that can be tested by data
	extrapolate	trends, cause/effect, impact of a decision
	predict	data, trends, inferences
	evaluate	text, images, points of view, solutions, phenomenon, graphics
	test	validity of assumptions, ideas, procedures, strategies
	argue	trends, cause/effect, strengths and weaknesses
	reflect	on strengths and weaknesses
	synthesise	data and knowledge, points of view from several sources
	analyse	text, images, graphs, data, points of view
	examine	data, visual images, arguments, points of view
	investigate	issues, problems
organise, sequence and explain	sequence	text, data, relationships, arguments, patterns
	visualise	trends, futures, patterns, cause and effect
	compare/contrast	data, visual images, arguments, points of view
	discuss	issues, data, relationships, choices/options
	interpret	symbols, text, images, graphs
	explain	explicit/implicit assumptions, bias, themes/arguments, cause/effect, strengths/weaknesses
	translate	data, visual images, arguments, points of view
	assess	probabilities, choices/options
	select	main points, words, ideas in text
identify, summarise and plan	reproduce	information, data, words, images, graphics
	respond	data, visual images, arguments, points of view
	relate	events, processes, situations
	demonstrate	probabilities, choices/options
	describe	data, visual images, arguments, points of view
	plan	strategies, ideas in text, arguments
	classify	information, data, words, images
	identify	spatial relationships, patterns, interrelationships
	summarise	main points, words, ideas in text, review, draft and edit

Appendix D – Glossary of Verbs

Verbs	Definition
Analyse	Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences
Apply	Use, utilise or employ in a particular situation
Argue	Give reasons for or against something
Assess	Make a judgement about the value of
Classify	Arrange into named categories in order to sort, group or identify
Compare	Estimate, measure or note how things are similar or dissimilar
Compose	The activity that occurs when students produce written, spoken, or visual texts
Contrast	Compare in such a way as to emphasise differences
Create	Bring into existence, to originate
Critically analyse	Analysis that engages with criticism and existing debate on the issue
Demonstrate	Give a practical exhibition an explanation
Describe	Give an account of characteristics or features
Discuss	Talk or write about a topic, taking into account different issues or ideas
Evaluate	Examine and judge the merit or significance of something
Examine	Determine the nature or condition of
Explain	Provide additional information that demonstrates understanding of reasoning and /or application
Extrapolate	Infer from what is known
Hypothesise	Put forward a supposition or conjecture to account for certain facts and used as a basis for further investigation by which it may be proved or disproved
Identify	Recognise and name
Interpret	Draw meaning from
Investigate	Planning, inquiry into and drawing conclusions about
Justify	Show how argument or conclusion is right or reasonable
Manipulate	Adapt or change
Plan	Strategize, develop a series of steps, processes
Predict	Suggest what might happen in the future or as a consequence of something
Reflect	The thought process by which students develop an understanding and appreciation of their own learning. This process draws on both cognitive and affective experience
Relate	Tell or report about happenings, events or circumstances
Represent	Use words, images, symbols or signs to convey meaning
Reproduce	Copy or make close imitation
Respond	React to a person or text
Select	Choose in preference to another or others
Sequence	Arrange in order
Summarise	Give a brief statement of the main points
Synthesise	Combine elements (information/ideas/components) into a coherent whole
Test	Examine qualities or abilities
Translate	Express in another language or form, or in simpler terms
Visualise	The ability to decode, interpret, create, question, challenge and evaluate texts that communicate with visual images as well as, or rather than, words

Appendix E – Glossary for ACT Senior Secondary Curriculum

Courses will detail what teachers are expected to teach and students are expected to learn for year 11 and 12. They will describe the knowledge, understanding and skills that students will be expected to develop for each learning area across the years of schooling.

Learning areas are broad areas of the curriculum, including English, mathematics, science, the arts, languages, health and physical education.

A **subject** is a discrete area of study that is part of a learning area. There may be one or more subjects in a single learning area.

Frameworks are system documents for Years 11 and 12 which provide the basis for the development and accreditation of any course within a designated learning area. In addition, frameworks provide a common basis for assessment, moderation and reporting of student outcomes in courses based on the framework.

The **course** sets out the requirements for the implementation of a subject. Key elements of a course include the rationale, goals, content descriptions, assessment, and achievement standards as designated by the framework.

BSSS courses will be organised into units. A unit is a distinct focus of study within a course. A standard 1.0 unit is delivered for a minimum of 55 hours generally over one semester.

Core units are foundational units that provide students with the breadth of the subject.

Additional units are avenues of learning that cannot be provided for within the four core 1.0 standard units by an adjustment to the program of learning.

An **Independent Study unit** is a pedagogical approach that empowers students to make decisions about their own learning. Independent Study units can be proposed by a student and negotiated with their teacher but must meet the specific unit goals and content descriptions as they appear in the course.

An **elective** is a lens for demonstrating the content descriptions within a standard 1.0 or half standard 0.5 unit.

A **lens** is a particular focus or viewpoint within a broader study.

Content descriptions refer to the subject-based knowledge, understanding and skills to be taught and learned.

A **program of learning** is what a college develops to implement the course for a subject and to ensure that the content descriptions are taught and learned.

Achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following completion of study of senior secondary course content for units in a subject.

ACT senior secondary system **curriculum** comprises all BSSS approved courses of study.

Appendix F – Glossary for Mathematical Methods

Unit 1

Functions and graphs

Asymptote

A line is an **asymptote** to a curve if the distance between the line and the curve approaches zero as they 'tend to infinity'. For example, the line with equation $x = \pi/2$ is a vertical asymptote to the graph of $y = \tan x$, and the line with equation $y = 0$ is a horizontal asymptote to the graph of $y = 1/x$.

Binomial distribution

The expansion $(x + y)^n = x^n + \binom{n}{1} x^{n-1} y + \cdots + \binom{n}{r} x^{n-r} y^r + \cdots + y^n$ is known as the **binomial theorem**. The numbers $\binom{n}{r} = \frac{n!}{r!(n-r)!} = \frac{n \times (n-1) \times \cdots \times (n-r+1)}{r \times (r-1) \times \cdots \times 2 \times 1}$ are called binomial coefficients.

Completing the square

The quadratic expression $ax^2 + bx + c$ can be rewritten as $a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$. Rewriting it in this way is called **completing the square**.

Discriminant

The **discriminant** of the quadratic expression $ax^2 + bx + c$ is the quantity $b^2 - 4ac$

Function

A **function** f is a rule that associates with each element x in a set S a unique element $f(x)$ in a set T. We write $x \longmapsto f(x)$ to indicate the mapping of x to $f(x)$. The set S is called the **domain** of f and the set T is called the **codomain**. The subset of T consisting of all the elements $f(x): x \in S$ is called the **range** of f. If we write $y = f(x)$ we say that x is the **independent variable** and y is the **dependent variable.**

Graph of a function

The **graph of a function** f is the set of all points (x, y) in Cartesian plane where x is in the domain of f and $y = f(x)$

Quadratic formula

If $ax^2 + bx + c = 0$ with $a \neq 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. This formula for the roots is called the **quadratic formula**.

Vertical line test

A relation between two real variables x and y is a function and $y = f(x)$ for some function f, if and only if each vertical line, i.e. each line parallel to the $y -$ axis, intersects the graph of the relation in at most one point. This test to determine whether a relation is, in fact, a function is known as the **vertical line test**.

Trigonometric functions

Circular measure is the measurement of angle size in radians.

Radian measure

The **radian measure** θ of an angle in a sector of a circle is defined by $\theta = \ell/r$, where r is the radius and ℓ is the arc length. Thus an angle whose degree measure is 180 has radian measure π.

Length of an arc

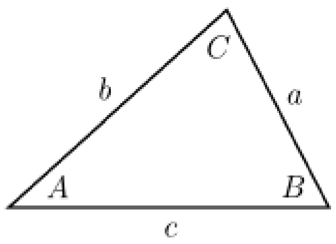
The **length of an arc in a circle** is given by $\ell = r\theta$, where ℓ is the arc length, r is the radius and θ is the angle subtended at the centre, measured in radians. This is simply a rearrangement of the formula defining the radian measure of an angle.

Sine rule and cosine rule

The lengths of the sides of a triangle are related to the sines of its angles by the equations

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

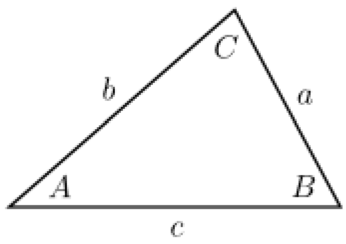
This is known as the **sine rule**.



The lengths of the sides of a triangle are related to the cosine of one of its angles by the equation

$$c^2 = a^2 + b^2 - 2ab\cos C$$

This is known as the **cosine rule**.



Sine and cosine functions

In the unit circle definition of cosine and sine, $\cos\theta$ and $\sin\theta$ are the x and y coordinates of the point on the unit circle corresponding to the angle θ

Period of a function

The period of a function $f(x)$ is the smallest positive number p with the property that $f(x+p) = f(x)$ for all x. The functions $\sin x$ and $\cos x$ both have period 2π and $\tan x$ has period π

Counting and Probability

Pascal's triangle

Pascal's triangle is a triangular arrangement of binomial coefficients. The n^{th} row consists of the **binomial coefficients** $\binom{n}{r}$, for $0 \le r \le n$, each interior entry is the sum of the two entries above it, and sum of the entries in the n^{th} row is 2^n

Conditional probability

The probability that an event A occurs can change if it becomes known that another event B occurs. The new probability is known as a **conditional probability** and is written as $P(A|B)$. If B has occurred, the sample space is reduced by discarding all outcomes that are not in the event B. The new sample space, called **the reduced sample space**, is B. The conditional probability of event A is given by $P(A|B) = \frac{P(A \cap B)}{P(B)}$.

Independent events

Two events are **independent** if knowing that one occurs tells us nothing about the other. The concept can be defined formally using probabilities in various ways: events A and B are independent if $P(A \cap B) = P(A)P(B)$, if $P(A|B) = P(A)$ or if $P(B) = P(B|A)$. For events A and B with non-zero probabilities, any one of these equations implies any other.

Mutually exclusive

Two events are **mutually exclusive** if there is no outcome in which both events occur.

Point and interval estimates

In statistics estimation is the use of information derived from a sample to produce an estimate of an unknown probability or population parameter. If the estimate is a single number, this number is called a **point estimate.** An **interval estimate** is an interval derived from the sample that, in some sense, is likely to contain the parameter.

A simple example of a point estimate of the probability p of an event is the relative frequency f of the event in a large number of Bernoulli trials. An example of an interval estimate for p is a confidence interval centred on the relative frequency f.

Relative frequency

If an event E occurs r times when a chance experiment is repeated n times, the **relative** frequency of E is r/n.

Unit 2

Exponential functions

Index laws

The index laws are the rules: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, and $(ab)^x = a^x b^x$, for any real numbers x, y, a and b, with $a > 0$ and $b > 0$

Algebraic properties of exponential functions

The algebraic properties of exponential functions are the index laws: $a^x a^y = a^{x+y}$, $a^{-x} = \frac{1}{a^x}$, $(a^x)^y = a^{xy}$, $a^0 = 1$, for any real numbers x, y, and a, with $a > 0$

Arithmetic and Geometric sequences and series

Arithmetic sequence

An arithmetic sequence is a sequence of numbers such that the difference of any two successive members of the sequence is a constant. For instance, the sequence

2, 5, 8, 11, 14, 17, …

is an arithmetic sequence with common difference 3.

If the initial term of an arithmetic sequence is a and the common difference of successive members is d, then the nth term t_n, of the sequence, is given by:

$t_n = a + (n - 1)d$ for $n \geq 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = t_n + d$ where d is the common difference and $n \geq 1$.

Geometric sequence

A geometric sequence is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed number called the **common ratio**. For example, the sequence

3, 6, 12, 24, ...

is a geometric sequence with common ratio 2. Similarly the sequence

40, 20, 10, 5, 2.5, …

is a geometric sequence with common ratio $\frac{1}{2}$.

If the initial term of a geometric sequence is a and the common ratio of successive members is r, then the n^{th} *term* t_n of the sequence, is given by:

$t_n = ar^{n-1}$ for $n \geq 1$

A recursive definition is

$t_1 = a$, $t_{n+1} = rt_n$ for $n \geq 1$ and where r is the constant ratio

Partial sums of a sequence (Series)

The sequence of partial sums of a sequence $t_1, \ldots, t_s$, … is defined by

$S_n = t_1 + \ldots + t_n$

Partial sum of an arithmetic sequence (Arithmetic series)

The partial sum S_n of the first n terms of an arithmetic sequence with first term a and common difference d.

$a, a + d, a + 2d, \ldots\ldots, a + (n - 1)d, \ldots$

is

$S_n = \frac{n}{2}(a + t_n) = \frac{n}{2}(2a + (n - 1)d)$ where t_n is the n^{th} term of the sequence.

The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$

Partial sums of a geometric sequence (Geometric series)

The partial sum S_n of the first n terms of a geometric sequence with first term a and common ratio r,

$a, ar, ar^2, \ldots\ldots, ar^{n-1}, \ldots$

is

$S_n = \frac{a(r^n - 1)}{r - 1}, r \neq 1.$

The partial sums form a sequence with $S_{n+1} = S_n + t_n$ and $S_1 = t_1$.

Introduction to differential calculus

Gradient (Slope)

The **gradient** of the straight line passing through points (x_1, y_1) and (x_2, y_2) is the ratio $\frac{y_2 - y_1}{x_2 - x_1}$.
Slope is a synonym for **gradient**.

Secant

A **secant** of the graph of a function is the straight line passing through two points on the graph. The line segment between the two points is called a **chord**.

Tangent line

The **tangent line** (or simply the **tangent**) to a curve at a given point P can be described intuitively as the straight line that "just touches" the curve at that point. At P where the tangent meets the curve the curve meet, the curve has "the same direction" as the tangent line. In this sense it is the best straight-line approximation to the curve at the point P.

Linearity property of the derivative

The **linearity property of the derivative** is summarized by the equations:

$\frac{d}{dx}(ky) = k\frac{dy}{dx}$ for any constant k

and $\frac{d}{dx}(y_1 + y_2) = \frac{dy_1}{dx} + \frac{dy_2}{dx}$

Local and global maximum and minimum

A **stationary point** on the graph $y = f(x)$ of a differentiable function is a point where $f'(x) = 0$.

We say that $f(x_0)$ is a **local maximum** of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a **global maximum** of the function $f(x)$ if $f(x) \leq f(x_0)$ for all values of x in the domain of f.

We say that $f(x_0)$ is a **local minimum** of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x near x_0. We say that $f(x_0)$ is a **global minimum** of the function $f(x)$ if $f(x) \geq f(x_0)$ for all values of x in the domain of f.

Unit 3

Further differentiation and applications

Euler's number

Euler's number e is an irrational number whose decimal expansion begins

$$e = 2.7182818284590452353602874713527 \cdots$$

It is the base of the natural logarithms, and can be defined in various ways including:

$e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + \cdots$ and $e = \lim_{n \to \infty} (1 + \frac{1}{n})^n$.

Product rule

The **product rule** relates the derivative of the product of two functions to the functions and their derivatives.

If $h(x) = f(x)g(x)$ then $h'(x) = f(x)g'(x) + f'(x)g(x)$,

and in Leibniz notation: $\frac{d}{dx}(uv) = u\frac{dv}{dx} + \frac{du}{dx}v$

Quotient rule

The **quotient rule** relates the derivative of the quotient of two functions to the functions and their derivatives

If $h(x) = \frac{f(x)}{g(x)}$ then $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{g(x)^2}$

and in Leibniz notation: $\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Composition of functions

If $y = g(x)$ and $z = f(y)$ for functions f and g, then z is a composite function of x.

We write $z = f \circ g(x) = f(g(x))$. For example, $z = \sqrt{x^2 + 3}$ expresses z as a composite of the functions $f(y) = \sqrt{y}$ and $g(x) = x^2 + 3$

Chain rule

The **chain rule** relates the derivative of the composite of two functions to the functions and their derivatives.

If $h(x) = f \circ g(x)$ then $(f \circ g)'(x) = f'\big(g(x)\big)g'(x)$,

and in Leibniz notation: $\frac{dz}{dx} = \frac{dz}{dy}\frac{dy}{dx}$

Concave up and concave down

A graph of $y = f(x)$ is concave up at a point P if points on the graph near P lie above the tangent at P. The graph is concave down at P if points on the graph near P lie below the tangent at P.

Point of inflection

A point P on the graph of $y = f(x)$ is a point of inflection if the concavity changes at P, i.e. points near P on one side of P lie above the tangent at P and points near P on the other side of P lie below the tangent at P

Second derivative test

According to the second derivative test, if $f'(x) = 0$, then $f(x)$ is a local maximum of f if $f''(x) < 0$ and $f(x)$ is a local minimum if $f''(x) > 0$

Integrals

Antidifferentiation

An **anti-derivative**, **primitive** or **indefinite integral** of a function $f(x)$ is a function $F(x)$ whose derivative is $f(x)$, i.e. $F'(x) = f(x)$.

The process of solving for anti-derivatives is called **anti-differentiation**.

Anti-derivatives are not unique. If $F(x)$ is an anti-derivative of $f(x)$, then so too is the function $F(x) + c$ where c is any number. We write $\int f(x)\,dx = F(x) + c$ to denote the set of all anti-derivatives of $f(x)$. The number c is called the **constant of integration**. For example, since $\frac{d}{dx}(x^3) = 3x^2$, we can write $\int 3x^2\,dx = x^3 + c$

The linearity property of anti-differentiation

The linearity property of anti-differentiation is summarized by the equations:

$\int kf(x)dx = k\int f(x)dx$ for any constant k and

$\int\big(f_1(x) + f_2(x)\big)dx = \int f_1(x)dx + \int f_2(x)\,dx$ for any two functions $f_1(x)$ and $f_2(x)$

Similar equations describe the linearity property of definite integrals:

$\int_a^b kf(x)dx = k\int_a^b f(x)dx$ for any constant k and

$\int_a^b\big(f_1(x) + f_2(x)\big)dx = \int_a^b f_1(x)dx + \int_a^b f_2(x)dx$ for any two functions $f_1(x)$ and $f_2(x)$

Additivity property of definite integrals

The **additivity property of definite integrals** refers to 'addition of intervals of integration':

$\int_a^b f(x)dx + \int_b^c f(x)dx = \int_a^c f(x)dx$ for any numbers a, b and c and any function $f(x)$.

The fundamental theorem of calculus

The **fundamental theorem of calculus** relates differentiation and definite integrals. It has two forms: $\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$ and $\int_a^b f'(x)dx = f(b) - f(a)$

Discrete random variables

Random variable

A **random variable** is a numerical quantity whose value depends on the outcome of a chance experiment. Typical examples are the number of people who attend an AFL grand final, the proportion of heads observed in 100 tosses of a coin, and the number of tonnes of wheat produced in Australia in a year.

A **discrete random variable** is one whose possible values are the counting numbers $0,1,2,3,\cdots$, or form a finite set, as in the first two examples.

A **continuous random variable** is one whose set of possible values are all of the real numbers in some interval.

Probability distribution

The **probability distribution** of a discrete random variable is the set of probabilities for each of its possible values.

Uniform discrete random variable

A **uniform discrete random variable** is one whose possible values have equal probability of occurrence. If there are n possible values, the probability of occurrence of any one of them is $1/n$.

Expected value

The **expected value** $E(X)$ of a random variable X is a measure of the central tendency of its distribution.

If X is discrete, $E(X) = \sum_i p_i x_i$, where the x_i are the possible values of X and

$p_i = P(X = x_i)$.

If X is continuous, $E(x) = \int_{-\infty}^{\infty} xp(x)dx$, where $p(x)$ is the probability density function of X

Mean of a random variable

The **mean** of a random variable is another name for its expected value.

Variance of a random variable

The **variance** $Var(X)$ of a random variable X is a measure of the 'spread' of its distribution.

If X is discrete, $Var(X) = \sum_i p_i (x_i - \mu)^2$, where $\mu = E(X)$ is the expected value.

If X is continuous, $Var(X) = \int_{-\infty}^{\infty} (x - \mu)^2 p(x)dx$

Standard deviation of a random variable

The **standard deviation** of a random variable is the square root of its variance.

Effect of linear change

The **effects of linear changes of scale and origin** on the mean and variance of a random variable are summarized as follows:

If X is a random variable and $Y = aX + b$, where a and b are constants, then

$E(Y) = aE(X) + b$ and $Var(Y) = a^2Var(X)$

Bernoulli random variable

A Bernoulli random variable has two possible values, namely 0 and 1. The parameter associated with such a random variable is the probability p of obtaining a 1.

Bernoulli trial

A **Bernoulli trial** is a chance experiment with possible outcomes, typically labeled 'success' and failure'.

Unit 4

The logarithmic function

Algebraic properties of logarithms

The algebraic properties of logarithms are the rules: $\log_a(xy) = \log_a x + \log_a y$, $\log_a \frac{1}{x} = -\log_a x$, and $\log_a 1 = 0$, for any positive real numbers x, y and a

Continuous random variables and the normal distribution

Probability density function

The **probability density function** of a continuous random variable is a function that describes the relative likelihood that the random variable takes a particular value. Formally, if $p(x)$ is the probability density of the continuous random variable X, then the probability that X takes a value in some interval $[a, b]$ is given by $\int_a^b p(x)\, dx$.

Uniform continuous random variable

A **uniform continuous random variable** X is one whose probability density function $p(x)$ has constant value on the range of possible values of X. If the range of possible values is the interval $[a, b]$ then $p(x) = \frac{1}{b-a}$ if $a \le x \le b$ and $p(x) = 0$ otherwise.

Triangular continuous random variable

A **triangular continuous random variable** X is one whose probability density function $p(x)$ has a graph with the shape of a triangle.

Quantile

A **quantile** t_α for a continuous random variable X is defined by $P(X > t_\alpha) = \alpha$, where $0 < \alpha < 1$.

The **median** m of X is the quantile corresponding to $\alpha = 0.5$: $P(X > m) = 0.5$

Interval estimates for proportions

Central limit theorem

There are various forms of the **Central limit theorem,** a result of fundamental importance in statistics. For the purposes of this course, it can be expressed as follows:

"If $\bar{X}$ is the mean of n independent values of random variable X which has a finite mean μ and a finite standard deviation σ, then as $n \to \infty$ the distribution of $\frac{\bar{X}-\mu}{\sigma/\sqrt{n}}$ approaches the standard normal distribution."

In the special case where X is a Bernoulli random variable with parameter p, $\bar{X}$ is the sample proportion $\hat{p}$, $\mu = p$ and $\sigma = \sqrt{p(1-p)}$. In this case the Central limit theorem is a statement that as $n \to \infty$ the distribution of $\frac{\hat{p}-p}{\sqrt{p(1-p)/n}}$ approaches the standard normal distribution.

Margin of error

The **margin of error** of a confidence interval of the form $f - E < p < f + E$ is E, the half-width of the confidence interval. It is the maximum difference between f and p if p is actually in the confidence interval.

Level of confidence

The **level of confidence** associated with a confidence interval for an unknown population parameter is the probability that a random confidence interval will contain the parameter.

Appendix G – Course Adoption

Conditions of Adoption

The course and units of this course are consistent with the philosophy and goals of the college and the adopting college has the human and physical resources to implement the course.

Adoption Process

Course adoption must be initiated electronically by an email from the principal or their nominated delegate to bssscertification@ed.act.edu.au. A nominated delegate must CC the principal.

The email will include the **Conditions of Adoption** statement above, and the table below adding the **College** name, and circling the **Classification/s** required.

College:	
Course Title:	**Mathematical Methods**
Classification/s:	**T**
Framework:	Mathematics 2020
Accredited from:	2014