



Mathematical Applications

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Cover Art provided by Canberra College student Aidan Giddings

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The ACT Senior Secondary System

The ACT senior secondary system recognises a range of university, vocational or life skills pathways.

The system is based on the premise that teachers are experts in their area: they know their students and community and are thus best placed to develop curriculum and assess students according to their needs and interests. Students have ownership of their learning and are respected as young adults who have a voice.

A defining feature of the system is school-based curriculum and continuous assessment. School-based curriculum provides flexibility for teachers to address students' needs and interests. College teachers have an opportunity to develop courses for implementation across ACT schools. Based on the courses that have been accredited by the BSSS, college teachers are responsible for developing programs of learning. A program of learning is developed by individual colleges to implement the courses and units they are delivering.

Teachers must deliver all content descriptions; however, they do have flexibility to emphasise some content descriptions over others. It is at the discretion of the teacher to select the texts or materials to demonstrate the content descriptions. Teachers can choose to deliver course units in any order and teach additional (not listed) content provided it meets the specific unit goals.

School-based continuous assessment means that students are continually assessed throughout years 11 and 12, with both years contributing equally to senior secondary certification. Teachers and students are positioned to have ownership of senior secondary assessment. The system allows teachers to learn from each other and to refine their judgement and develop expertise.

Senior secondary teachers have the flexibility to assess students in a variety of ways. For example: multimedia presentation, inquiry-based project, test, essay, performance and/or practical demonstration may all have their place. College teachers are responsible for developing assessment instruments with task specific rubrics and providing feedback to students.

The integrity of the ACT Senior Secondary Certificate is upheld by a robust, collaborative and rigorous structured consensus-based peer reviewed moderation process. System moderation involves all Year 11 and 12 teachers from public, non-government and international colleges delivering the ACT Senior Secondary Certificate.

Only students who desire a pathway to university are required to sit a general aptitude test, referred to as the ACT Scaling Test (AST), which moderates student course scores across subjects and colleges. Students are required to use critical and creative thinking skills across a range of disciplines to solve problems. They are also required to interpret a stimulus and write an extended response.

Senior secondary curriculum makes provision for student-centred teaching approaches, integrated and project-based learning inquiry, formative assessment and teacher autonomy. ACT Senior Secondary Curriculum makes provision for diverse learners and students with mild to moderate intellectual disabilities, so that all students can achieve an ACT Senior Secondary Certificate.

The ACT Board of Senior Secondary Studies (BSSS) leads senior secondary education. It is responsible for quality assurance in senior secondary curriculum, assessment and certification. The Board consists of representatives from colleges, universities, industry, parent organisations and unions. The Office of the Board of Senior Secondary Studies (OBSSS) consists of professional and administrative staff who support the Board in achieving its objectives and functions.

ACT Senior Secondary Certificate

Courses of study for the ACT Senior Secondary Certificate:

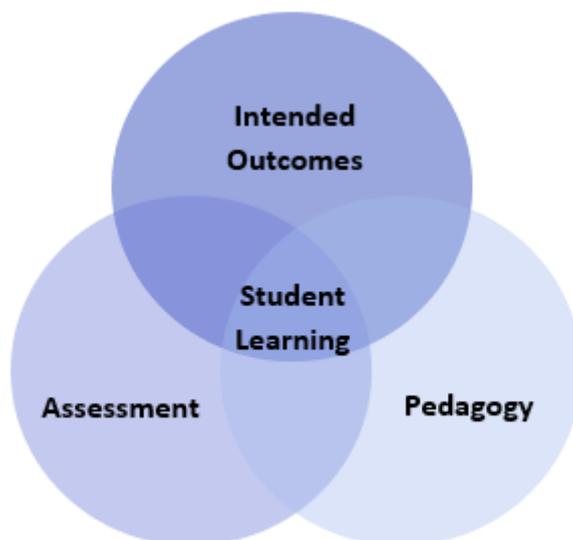
- provide a variety of pathways, to meet different learning needs and encourage students to complete their secondary education
- enable students to develop the essential capabilities for twenty-first century learners
- empower students as active participants in their own learning
- engage students in contemporary issues relevant to their lives
- foster students' intellectual, social and ethical development
- nurture students' wellbeing, and physical and spiritual development
- enable effective and respectful participation in a diverse society.

Each course of study:

- comprises an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum
- is based on a model of learning that integrates intended student outcomes, pedagogy and assessment
- outlines teaching strategies which are grounded in learning principles and encompass quality teaching
- promotes intellectual quality, establish a rich learning environment and generate relevant connections between learning and life experiences
- provides formal assessment and certification of students' achievements.

Underpinning beliefs

- All students are able to learn.
- Learning is a partnership between students and teachers.
- Teachers are responsible for advancing student learning.



Learning Principles

1. Learning builds on existing knowledge, understandings and skills.
(Prior knowledge)
2. When learning is organised around major concepts, principles and significant real world issues, within and across disciplines, it helps students make connections and build knowledge structures.
(Deep knowledge and connectedness)
3. Learning is facilitated when students actively monitor their own learning and consciously develop ways of organising and applying knowledge within and across contexts.
(Metacognition)
4. Learners' sense of self and motivation to learn affects learning.
(Self-concept)
5. Learning needs to take place in a context of high expectations.
(High expectations)
6. Learners learn in different ways and at different rates.
(Individual differences)
7. Different cultural environments, including the use of language, shape learners' understandings and the way they learn.
(Socio-cultural effects)
8. Learning is a social and collaborative function as well as an individual one.
(Collaborative learning)
9. Learning is strengthened when learning outcomes and criteria for judging learning are made explicit and when students receive frequent feedback on their progress.
(Explicit expectations and feedback)

General Capabilities

All courses of study for the ACT Senior Secondary Certificate should enable students to develop essential capabilities for twenty-first century learners. These 'capabilities' comprise an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum.

The capabilities include:

- literacy
- numeracy
- information and communication technology (ICT)
- critical and creative thinking
- personal and social
- ethical behaviour
- intercultural understanding

Courses of study for the ACT Senior Secondary Certificate should be both relevant to the lives of students and incorporate the contemporary issues they face. Hence, courses address the following three priorities. These priorities are:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability

Elaboration of these General Capabilities and priorities is available on the ACARA website at www.australiancurriculum.edu.au.

Literacy in Mathematics

In the senior years, literacy skills and strategies enable students to express, interpret and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their abilities to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will develop their numeracy skills at a more sophisticated level than in Foundation to Year 10. This subject contains financial applications of mathematics that will assist students to become literate consumers of investments, loans and superannuation products. It also contains statistics topics that will equip students for the ever-increasing demands of the information age.

Information and Communication Technology (ICT) Capability in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, data representation and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, using data, addressing problems, and operating systems in authentic situations.

Critical and Creative Thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions do not match, it is due to a flaw in the theory or in the method of applying the theory to make predictions, or both. They revise, or reapply, their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and in making accurate predictions.

Personal and Social Capability in Mathematics

In the senior years students develop personal and social competence in mathematics by setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision making.

The elements of personal and social competence relevant to mathematics mainly include the application of mathematical skills for decision making, life-long learning, citizenship and self-management. As part of their mathematical explorations and investigations, students work collaboratively in teams, as well as independently.

Ethical Understanding in Mathematics

In the senior years students develop ethical understanding in mathematics through decision making connected with ethical dilemmas that arise when engaged in mathematical calculation, the dissemination of results, and the social responsibility associated with teamwork and attribution of input.

The areas relevant to mathematics include issues associated with ethical decision making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and the examined ethical behaviour. Students develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural Understanding in Mathematics

Students understand mathematics as a socially constructed body of knowledge that uses universal symbols but has its origins in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number, and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Cross-Curriculum Priorities

Aboriginal and Torres Strait Islander Histories and Cultures

The Senior Secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

Asia and Australia's Engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss this global issue and learn the importance of respecting and valuing a wide range of world perspectives.

Mathematical Applications T

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe many aspects of the world in the twenty-first century. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve practical problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Mathematical Applications is designed for those students who want to extend their mathematical skills beyond Year 10 level but whose future studies or employment pathways do not require knowledge of calculus. The subject is designed for students who have a wide range of educational and employment aspirations, including continuing their studies at university or TAFE.

The proficiency strands of the F-10 curriculum – Understanding, Fluency, Problem solving and Reasoning – are still relevant and are inherent in all aspects of this subject. Each of these proficiencies is essential, and all are mutually reinforcing. Fluency, for example, might include learning to perform routine calculations efficiently and accurately, or being able to recognise quickly from a problem description the appropriate mathematical process or model to apply.

Understanding, furthermore, that a single mathematical process can be used in seemingly different situations, helps students to see the connections between different areas of study and encourages the transfer of learning. This is an important part of learning the art of mathematical problem solving. In performing such analyses, reasoning is required at each decision-making step and in drawing appropriate conclusions. Presenting the analysis in a logical and clear manner to explain the reasoning used is also an integral part of the learning process.

Throughout the subject there is also an emphasis on the use and application of digital technologies.

Goals

Mathematical Applications aims to develop students’:

- understanding of concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- ability to solve applied problems using concepts and techniques drawn from the topic areas of number and algebra, geometry and trigonometry, graphs and networks, and statistics
- reasoning and interpretive skills in mathematical and statistical contexts
- capacity to communicate the results of a mathematical or statistical problem-solving activity in a concise and systematic manner using appropriate mathematical and statistical language
- capacity to choose and use technology appropriately and efficiently.

Student Group

Links to Foundation to Year 10

The Mathematical Applications subject provides students with a breadth of mathematical and statistical experience that encompasses and builds on all three strands of the F-10 curriculum.

Unit Titles

- Unit 1: Mathematical Applications
- Unit 2: Mathematical Applications
- Unit 3: Mathematical Applications
- Unit 4: Mathematical Applications

Organisation of Content

Mathematical Applications focuses on the use of mathematics to solve problems in contexts that involve financial modelling, geometric and trigonometric analysis, graphical and network analysis, and growth and decay in sequences. It also provides opportunities for students to develop systematic strategies based on the statistical investigation process for answering statistical questions that involve analysing univariate and bivariate data, including time series data.

Mathematical Applications is organised into four units. The topics in each unit broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The units provide a blending of algebraic, geometric and statistical thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction.

| | Unit 1 | Unit 2 | Unit 3 | Unit 4 |
|----------------------------------|--|---|---|---|
| Mathematical Applications | <ul style="list-style-type: none"> • consumer arithmetic • algebra and matrices • shape and measurement | <ul style="list-style-type: none"> • univariate data analysis and the statistical investigation process • applications of trigonometry • linear equations and their graphs | <ul style="list-style-type: none"> • bivariate data analysis • growth and decay in sequences • graphs and networks | <ul style="list-style-type: none"> • time series analysis • loans, investments and annuities • networks and decision mathematics |

Unit 1: Mathematical Applications

Unit 1 has three topics: 'Consumer arithmetic', 'Algebra and matrices', and 'Shape and measurement'. 'Consumer arithmetic' reviews the concepts of rate and percentage change in the context of earning and managing money, and provides fertile ground for the use of spreadsheets. 'Algebra and matrices' continues the F-10 study of algebra and introduces the new topic of matrices. 'Shape and measurement' extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Unit 2: Mathematical Applications

Unit 2 has three topics: 'Univariate data analysis and the statistical investigation process', 'Linear equations and their graphs', and 'Applications of trigonometry'. 'Univariate data analysis and the statistical investigation process' develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation. 'Applications of trigonometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression, and bearings in navigation. 'Linear equations and their graphs' uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations.

Unit 3: Mathematical Applications

Unit 3 has three topics: 'Bivariate data analysis', 'Growth and decay in sequences', and 'Graphs and networks'. 'Bivariate data analysis' introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including using the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process. 'Growth and decay in sequences' employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4. 'Graphs and networks' introduces students to the language of graphs and the way in which graphs, represented as a collection of points and interconnecting lines, can be used to analyse everyday situations such as a rail or social network.

Unit 4: Mathematical Applications

Unit 4 has three topics: 'Time series analysis', 'Loans, investments and annuities', and 'Networks and decision mathematics'. 'Time series analysis' continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process. 'Loans and investments' aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments. 'Networks and decision mathematics' uses networks to model and aid decision making in practical situations.

Assessment

The identification of criteria within the achievement standards and assessment task types and weightings provides a common and agreed basis for the collection of evidence of student achievement.

Assessment Criteria (the dimensions of quality that teachers look for in evaluating student work) provide a common and agreed basis for judgement of performance against unit and course goals, within and across colleges. Over a course, teachers must use all these criteria to assess students' performance but are not required to use all criteria on each task. Assessment criteria are to be used holistically on a given task and in determining the unit grade.

Assessment Tasks elicit responses that demonstrate the degree to which students have achieved the goals of a unit based on the assessment criteria. The Common Curriculum Elements (CCE) is a guide to developing assessment tasks that promote a range of thinking skills (see Appendix C). It is highly desirable that assessment tasks engage students in demonstrating higher order thinking.

Rubrics are constructed for individual tasks, informing the assessment criteria relevant for a particular task and can be used to assess a continuum that indicates levels of student performance against each criterion.

Assessment Criteria

Students will be assessed on the degree to which they demonstrate an understanding of:

- concepts and techniques
- reasoning and communications.

Assessment Task Types

Suggested tasks:

- project/assignment
- modelling projects
- portfolio
- journal
- validation activity
- presentation such as a pitch, poster, vodcast, interview
- practical activity such as a demonstration
- test/examination
- online adaptive tasks/quiz

Weightings in T - 1.0 Units:

No task to be weighted more than 50% for a standard 1.0 unit.

Additional Assessment Information

Requirements

- For a standard unit (1.0), students must complete a minimum of three assessment tasks and a maximum of five.
- For a half standard unit (0.5), students must complete a minimum of two and a maximum of three assessment tasks.
- Students should experience a variety of task types (test and non-test) and different modes of communication to demonstrate the Achievement Standards.
- Students are required to undertake at least one problem solving investigation task each semester. This task may be completed individually or collaboratively. They are required to plan, enquire into and draw conclusions about key unit concepts. Students may respond in forms such as modelling projects, problem solving and practical activities.
- Assessment tasks for a standard (1.0) or half-standard (0.5) unit must be informed by the Achievement Standards.

Advice

- It is recommended that the total component of unsupervised tasks be no greater than 30%.
- For tasks completed in unsupervised conditions, schools need to have mechanisms to uphold academic integrity, for example, student declaration, plagiarism software, oral defence, interview, other validation tasks

Achievement Standards

Years 11 and 12 achievement standards are written for A/T courses. A single achievement standard is written for M courses.

A Year 12 student in any unit is assessed using the Year 12 achievement standards. A Year 11 student in any unit is assessed using the Year 11 achievement standards. Year 12 achievement standards reflect higher expectations of student achievement compared to the Year 11 achievement standards. Years 11 and 12 achievement standards are differentiated by cognitive demand, the number of dimensions and the depth of inquiry.

An achievement standard cannot be used as a rubric for an individual assessment task. Assessment is the responsibility of the college. Student tasks may be assessed using rubrics or marking schemes devised by the college. A teacher may use the achievement standards to inform development of rubrics. The verbs used in achievement standards may be reflected in the rubric. In the context of combined Years 11 and 12 classes, it is best practice to have a distinct rubric for Years 11 and 12. These rubrics should be available for students prior to completion of an assessment task so that success criteria are clear.

Student achievement in A, T and M units is reported based on system standards as an A-E grade. Grade descriptors and standard work samples where available, provide a guide for teacher judgement of students' achievement over the unit.

Grades are awarded on the proviso that the assessment requirements have been met. Teachers will consider, when allocating grades, the degree to which students demonstrate their ability to complete and submit tasks within a specified time frame.

Achievement Standards for Mathematics T Course – Year 11

| | <i>A student who achieves an A grade typically</i> | <i>A student who achieves a B grade typically</i> | <i>A student who achieves a C grade typically</i> | <i>A student who achieves a D grade typically</i> | <i>A student who achieves an E grade typically</i> |
|-------------------------------------|---|--|--|---|---|
| Concepts and Techniques | <ul style="list-style-type: none"> critically applies mathematical concepts in a variety of complex contexts to routine and non-routine problems selects and applies advanced mathematical techniques to solve complex problems in a variety of contexts constructs, selects and applies complex mathematical models to routine and non-routine problems in a variety of contexts uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts | <ul style="list-style-type: none"> applies mathematical concepts in a variety of contexts to routine and non-routine problems selects and applies mathematical techniques to solve routine and non-routine problems in a variety of contexts selects and applies mathematical models to routine and non-routine problems to a variety of contexts uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts | <ul style="list-style-type: none"> applies mathematical concepts in some contexts to routine and non-routine problems applies mathematical techniques to solve routine and non-routine problems in some contexts applies mathematical models to routine and non-routine problems in some contexts uses digital technologies appropriately to solve routine and non-routine problems in some contexts | <ul style="list-style-type: none"> applies simple mathematical concepts in limited contexts to routine problems applies simple mathematical techniques to solve routine problems in limited contexts applies simple mathematical models to routine problems in limited contexts uses digital technologies appropriately to solve routine problems in limited contexts | <ul style="list-style-type: none"> applies simple mathematical concepts in structured contexts uses simple mathematical techniques to solve routine problems in structured contexts demonstrates limited familiarity with mathematical models in structured contexts uses digital technologies to solve routine problems in structured contexts |
| Reasoning and Communications | <ul style="list-style-type: none"> represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and well-reasoned, using appropriate and accurate language evaluates the reasonableness of solutions to routine and non-routine problems in a variety of contexts reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively evaluates the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems a variety of contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language analyses the reasonableness of solutions to routine and non-routine problems reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively analyses the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents mathematical concepts in numerical, graphical and symbolic form to some routine and some non-routine problems in some contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language explains the reasonableness of solutions to some routine and non-routine problems reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively explains the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in limited contexts communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language describes the appropriateness of solutions to routine problems reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively describes the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents simple mathematical concepts in numerical, graphical or symbolic form in structured contexts communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language identifies solutions to routine problems in structured contexts reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively identifies some ways in which Mathematics is used to generate knowledge in the public good |

Achievement Standards for Mathematics T Course – Year 12

| | <i>A student who achieves an A grade typically</i> | <i>A student who achieves a B grade typically</i> | <i>A student who achieves a C grade typically</i> | <i>A student who achieves a D grade typically</i> | <i>A student who achieves an E grade typically</i> |
|-------------------------------------|--|---|--|--|--|
| Concepts and Techniques | <ul style="list-style-type: none"> critically and creatively applies mathematical concepts in a variety of complex contexts to routine and non-routine problems synthesises information to select and apply mathematical techniques to solve complex problems in a variety of contexts constructs, selects and applies mathematical models to a variety of contexts in routine and non-routine problems uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts | <ul style="list-style-type: none"> critically applies mathematical concepts in a variety of contexts to routine and non-routine problems analyses information to select and apply mathematical techniques to solve routine and non-routine problems in a variety of contexts selects and applies mathematical models to routine and non-routine problems in a variety of contexts uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts | <ul style="list-style-type: none"> applies mathematical concepts in some contexts to routine and non-routine problems selects and applies mathematical techniques to solve routine and some non-routine problems in some contexts applies mathematical models to routine and non-routine problems in some contexts uses digital technologies appropriately to solve routine and non-routine problems in a variety of contexts | <ul style="list-style-type: none"> applies simple mathematical concepts in limited contexts to routine problems applies simple mathematical techniques to solve routine problems in limited contexts applies simple mathematical models to routine problems in limited contexts uses digital technologies appropriately to solve routine problems in limited contexts | <ul style="list-style-type: none"> applies simple mathematical concepts in structured contexts uses simple mathematical techniques to solve routine problems in structured contexts demonstrates limited familiarity with mathematical models to solve routine problems in structured contexts uses digital technologies to solve routine problems in structured contexts |
| Reasoning and Communications | <ul style="list-style-type: none"> represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and reasoned, using appropriate and accurate language evaluates the solutions to routine and non-routine problems in a variety of contexts evaluates methods and models for their strengths and limitations when developing solutions to routine and non-routine problems reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively evaluates the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language analyses the solutions to routine and non-routine problems in some contexts analyses strengths and limitations of models used when developing solutions to routine and non-routine problems reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively analyses the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents mathematical concepts in numerical, graphical and symbolic form in some routine and non-routine problems in some contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language explains solutions to some routine and non-routine problems in some contexts explains strengths and limitations of models used when developing solutions to some routine and non-routine problems reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively explains the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in structured contexts communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language describes solutions to routine problems in limited contexts describes strengths or limitations of simple models when solving routine problems reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively describes the potential of Mathematics to generate knowledge in the public good | <ul style="list-style-type: none"> represents simple mathematical concepts in numerical, graphical or symbolic form in simple problems in structured contexts communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language identifies solutions to routine problems in structured contexts identifies strengths or limitations of simple models in relation to routine problems reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively identifies some ways in which Mathematics is used to generate knowledge in the public good |

Unit 1: Mathematical Applications

Value: 1.0

Unit 1a: Mathematical Applications

Value: 0.5

Unit 1b: Mathematical Applications

Value: 0.5

Unit Description

This unit has three topics: 'Consumer arithmetic', 'Algebra and matrices', and 'Shape and measurement'.

'Consumer arithmetic' reviews the concepts of rate and percentage change in the context of earning and managing money, and provides a fertile ground for the use of spreadsheets.

'Algebra and matrices' continues the F-10 study of algebra and introduces the new topic of matrices.

'Shape and measurement' builds on and extends the knowledge and skills students developed in the F-10 curriculum with the concept of similarity and associated calculations involving simple and compound geometric shapes. The emphasis in this topic is on applying these skills in a range of practical contexts, including those involving three-dimensional shapes.

Classroom access to the technology necessary to support the computational aspects of the topics in this unit is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques introduced in consumer arithmetic, algebra and matrices, and shape and measurement
- apply reasoning skills and solve practical problems arising in consumer arithmetic, algebra and matrices, and shape and measurement
- communicate their arguments and strategies, when solving problems, using appropriate mathematical language
- interpret mathematical information, and ascertain the reasonableness of their solutions to problems
- choose and use technology appropriately and efficiently

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Consumer arithmetic

Applications of rates and percentages:

- review rates and percentages
- calculate weekly or monthly wage from an annual salary, wages from an hourly rate including situations involving overtime and other allowances and earnings based on commission or piecework
- calculate payments based on government allowances and pensions
- prepare a personal budget for a given income taking into account fixed and discretionary spending
- compare prices and values using the unit cost method
- apply percentage increase or decrease in various contexts; for example, determining the impact of inflation on costs and wages over time, calculating percentage mark-ups and discounts, calculating GST, calculating profit or loss in absolute and percentage terms, and calculating simple and compound interest
- use currency exchange rates to determine the cost in Australian dollars of purchasing a given amount of a foreign currency, such as US\$1500, or the value of a given amount of foreign currency when converted to Australian dollars, such as the value of €2050 in Australian dollars
- calculate the dividend paid on a portfolio of shares, given the percentage dividend or dividend paid per share, for each share; and compare share values by calculating a price-to-earnings ratio.

Use of spreadsheets:

- use a spreadsheet to display examples of the above computations when multiple or repeated computations are required; for example, preparing a wage-sheet displaying the weekly earnings of workers in a fast food store where hours of employment and hourly rates of pay may differ, preparing a budget, or investigating the potential cost of owning and operating a car over a year.

Topic 2: Algebra and matrices

Linear and non-linear expressions:

- substitute numerical values into linear algebraic and simple non-linear algebraic expressions, and evaluate
- find the value of the subject of the formula, given the values of the other pronumerals in the formula
- use a spreadsheet or an equivalent technology to construct a table of values from a formula, including two-by-two tables for formulas with two variable quantities; for example, a table displaying the body mass index (BMI) of people of different weights and heights

Matrices and matrix arithmetic:

- use matrices for storing and displaying information that can be presented in rows and columns; for example, databases, links in social or road networks
- recognise different types of matrices (row, column, square, zero, identity) and determine their size

- perform matrix addition, subtraction, multiplication by a scalar, and matrix multiplication, including determining the power of a matrix using technology with matrix arithmetic capabilities when appropriate
- use matrices, including matrix products and powers of matrices, to model and solve problems; for example, costing or pricing problems, squaring a matrix to determine the number of ways pairs of people in a communication network can communicate with each other via a third person

Topic 3: Shape and measurement

Pythagoras' Theorem:

- review Pythagoras' Theorem and use it to solve practical problems in two dimensions and for simple applications in three dimensions

Mensuration:

- solve practical problems requiring the calculation of perimeters and areas of circles, sectors of circles, triangles, rectangles, parallelograms and composites
- calculate the volumes of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the volume of water contained in a swimming pool
- calculate the surface areas of standard three-dimensional objects such as spheres, rectangular prisms, cylinders, cones, pyramids and composites in practical situations; for example, the surface area of a cylindrical food container

Similar figures and scale factors:

- review the conditions for similarity of two-dimensional figures including similar triangles
- use the scale factor for two similar figures to solve linear scaling problems
- obtain measurements from scale drawings, such as maps or building plans, to solve problems
- obtain a scale factor and use it to solve scaling problems involving the calculation of the areas of similar figures
- obtain a scale factor and use it to solve scaling problems involving the calculation of surface areas and volumes of similar solids

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasize some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 10-12.

Unit 2: Mathematical Applications

Value: 1.0

Unit 2a: Mathematical Applications

Value: 0.5

Unit 2b: Mathematical Applications

Value: 0.5

Unit Description

This unit has three topics: 'Univariate data analysis and the statistical investigation process', 'Linear equations and their graphs'; and 'Applications of trigonometry'.

'Univariate data analysis and the statistical investigation process' develops students' ability to organise and summarise univariate data in the context of conducting a statistical investigation.

'Linear equations and their graphs' uses linear equations and straight-line graphs, as well as linear-piecewise and step graphs, to model and analyse practical situations.

'Applications of trigonometry' extends students' knowledge of trigonometry to solve practical problems involving non-right-angled triangles in both two and three dimensions, including problems involving the use of angles of elevation and depression and bearings in navigation.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in univariate data analysis and the statistical investigation process, linear equations and their graphs, and applications of trigonometry
- apply reasoning skills and solve practical problems in univariate data analysis and the statistical investigation process, linear equations and their graphs, and the applications of trigonometry
- implement the statistical investigation process in contexts requiring the analysis of univariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Univariate data analysis and the statistical investigation process

The statistical investigation process:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results

Making sense of data relating to a single statistical variable:

- classify a categorical variable as ordinal, such as income level (high, medium, low), or nominal, such as place of birth (Australia, overseas), and use tables and bar charts to organise and display the data
- classify a numerical variable as discrete, such as the number of rooms in a house, or continuous, such as the temperature in degrees Celsius
- with the aid of an appropriate graphical display (chosen from dot plot, stem plot, bar chart or histogram), describe the distribution of a numerical dataset in terms of modality (uni or multimodal), shape (symmetric versus positively or negatively skewed), location and spread and outliers, and interpret this information in the context of the data
- determine the mean and standard deviation of a dataset and use these statistics as measures of location and spread of a data distribution, being aware of their limitations

Comparing data for a numerical variable across two or more groups:

- construct and use parallel box plots (including the use of the 'Q1 – 1.5 x IQR' and 'Q3 + 1.5 x IQR' criteria for identifying possible outliers) to compare groups in terms of location (median), spread (IQR and range) and outliers and to interpret and communicate the differences observed in the context of the data
- compare groups on a single numerical variable using medians, means, IQRs, ranges or standard deviations, as appropriate; interpret the differences observed in the context of the data; and report the findings in a systematic and concise manner
- implement the statistical investigation process to answer questions that involve comparing the data for a numerical variable across two or more groups; for example, are Year 11 students the fittest in the school?

Topic 2: Applications of trigonometry

Applications of trigonometry:

- review the use of the trigonometric ratios to find the length of an unknown side or the size of an unknown angle in a right-angled triangle
- determine the area of a triangle given two sides and an included angle by using the rule $Area = \frac{1}{2}ab \sin C$, or given three sides by using Heron's rule, and solve related practical problems
- solve problems involving non-right-angled triangles using the sine rule (ambiguous case excluded) and the cosine rule
- solve practical problems involving the trigonometry of right-angled and non-right-angled triangles, including problems involving angles of elevation and depression and the use of bearings in navigation

Topic 3: Linear equations and their graphs

Linear equations:

- identify and solve linear equations
- develop a linear formula from a word description

Straight-line graphs and their applications:

- construct straight-line graphs both with and without the aid of technology
- determine the slope and intercepts of a straight-line graph from both its equation and its plot
- interpret, in context, the slope and intercept of a straight-line graph used to model and analyse a practical situation
- construct and analyse a straight-line graph to model a given linear relationship; for example, modelling the cost of filling a fuel tank of a car against the number of litres of petrol required

Simultaneous linear equations and their applications:

- solve a pair of simultaneous linear equations, using technology when appropriate
- solve practical problems that involve finding the point of intersection of two straight-line graphs; for example, determining the break-even point where cost and revenue are represented by linear equations

Piece-wise linear graphs and step graphs:

- sketch piece-wise linear graphs and step graphs, using technology when appropriate
- interpret piece-wise linear and step graphs used to model practical situations; for example, the tax paid as income increases, the change in the level of water in a tank over time when water is drawn off at different intervals and for different periods of time, the charging scheme for sending parcels of different weights through the post

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer pages 10-12.

Unit 3: Mathematical Applications

Value: 1.0

Unit 3a: Mathematical Applications

Value: 0.5

Unit 3b: Mathematical Applications

Value: 0.5

Prerequisites

Nil

Unit Description

This unit has three topics: 'Bivariate data analysis', 'Growth and decay in sequences' and 'Graphs and networks'.

'Bivariate data analysis' introduces students to some methods for identifying, analysing and describing associations between pairs of variables, including the use of the least-squares method as a tool for modelling and analysing linear associations. The content is to be taught within the framework of the statistical investigation process.

'Growth and decay in sequences' employs recursion to generate sequences that can be used to model and investigate patterns of growth and decay in discrete situations. These sequences find application in a wide range of practical situations, including modelling the growth of a compound interest investment, the growth of a bacterial population, or the decrease in the value of a car over time. Sequences are also essential to understanding the patterns of growth and decay in loans and investments that are studied in detail in Unit 4.

'Graphs and networks' introduces students to the language of graphs and the ways in which graphs, represented as a collection of points and interconnecting lines, can be used to model and analyse everyday situations such as a rail or social network.

Classroom access to technology to support the graphical and computational aspects of these topics is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in bivariate data analysis, growth and decay in sequences, and graphs and networks
- apply reasoning skills and solve practical problems in bivariate data analysis, growth and decay in sequences, and graphs and networks
- implement the statistical investigation process in contexts requiring the analysis of bivariate data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Bivariate data analysis

The statistical investigation process:

- review the statistical investigation process; for example, identifying a problem and posing a statistical question, collecting or obtaining data, analysing the data, interpreting and communicating the results

Identifying and describing associations between two categorical variables:

- construct two-way frequency tables and determine the associated row and column sums and percentages
- use an appropriately percentaged two-way frequency table to identify patterns that suggest the presence of an association
- describe an association in terms of differences observed in percentages across categories in a systematic and concise manner, and interpret this in the context of the data

Identifying and describing associations between two numerical variables:

- construct a scatterplot to identify patterns in the data suggesting the presence of an association
- describe an association between two numerical variables in terms of direction (positive/negative), form (linear/non-linear) and strength (strong/moderate/weak)
- calculate and interpret the correlation coefficient (r) to quantify the strength of a linear association

Fitting a linear model to numerical data:

- identify the response variable and the explanatory variable
- use a scatterplot to identify the nature of the relationship between variables
- model a linear relationship by fitting a least-squares line to the data
- use a residual plot to assess the appropriateness of fitting a linear model to the data
- interpret the intercept and slope of the fitted line
- use the coefficient of determination to assess the strength of a linear association in terms of the explained variation
- use the equation of a fitted line to make predictions
- distinguish between interpolation and extrapolation when using the fitted line to make predictions, recognising the potential dangers of extrapolation
- write up the results of the above analysis in a systematic and concise manner

Association and causation:

- recognise that an observed association between two variables does not necessarily mean that there is a causal relationship between them

- identify possible non-causal explanations for an association, including coincidence and confounding due to a common response to another variable, and communicate these explanations in a systematic and concise manner

The data investigation process:

- implement the statistical investigation process to answer questions that involve identifying, analysing and describing associations between two categorical variables or between two numerical variables; for example, is there an association between attitude to capital punishment (agree with, no opinion, disagree with) and sex (male, female)? is there an association between height and foot length?

Topic 2: Growth and decay in sequences

The arithmetic sequence:

- use recursion to generate an arithmetic sequence
- display the terms of an arithmetic sequence in both tabular and graphical form and demonstrate that arithmetic sequences can be used to model linear growth and decay in discrete situations
- deduce a rule for the n th term of a particular arithmetic sequence from the pattern of the terms in an arithmetic sequence, and use this rule to make predictions
- use arithmetic sequences to model and analyse practical situations involving linear growth or decay; for example, analysing a simple interest loan or investment, calculating a taxi fare based on the flag fall and the charge per kilometre, or calculating the value of an office photocopier at the end of each year using the straight-line method or the unit cost method of depreciation

The geometric sequence:

- use recursion to generate a geometric sequence
- display the terms of a geometric sequence in both tabular and graphical form and demonstrate that geometric sequences can be used to model exponential growth and decay in discrete situations
- deduce a rule for the n th term of a particular geometric sequence from the pattern of the terms in the sequence, and use this rule to make predictions
- use geometric sequences to model and analyse (numerically, or graphically only) practical problems involving geometric growth and decay; for example, analysing a compound interest loan or investment, the growth of a bacterial population that doubles in size each hour, the decreasing height of the bounce of a ball at each bounce; or calculating the value of office furniture at the end of each year using the declining (reducing) balance method to depreciate.

Sequences generated by first-order linear recurrence relations:

- use a general first-order linear recurrence relation to generate the terms of a sequence and to display it in both tabular and graphical form
- recognise that a sequence generated by a first-order linear recurrence relation can have a long term increasing, decreasing or steady-state solution
- use first-order linear recurrence relations to model and analyse (numerically or graphically only) practical problems; for example, investigating the growth of a trout population in a lake recorded at the end of each year and where limited recreational fishing is permitted, or the amount owing on a reducing balance loan after each payment is made

Topic 3: Graphs and networks

The definition of a graph and associated terminology:

- explain the meanings of the terms: graph, edge, vertex, loop, degree of a vertex, subgraph, simple graph, complete graph, bipartite graph, directed graph (digraph), arc, weighted graph, and network
- identify practical situations that can be represented by a network, and construct such networks; for example, trails connecting camp sites in a National Park, a social network, a transport network with one-way streets, a food web, the results of a round-robin sporting competition
- construct an adjacency matrix from a given graph or digraph

Planar graphs:

- explain the meaning of the terms: planar graph, and face
- apply Euler's formula, $v + f - e = 2$, to solve problems relating to planar graphs

Paths and cycles:

- explain the meaning of the terms: walk, trail, path, closed walk, closed trail, cycle, connected graph, and bridge
- investigate and solve practical problems to determine the shortest path between two vertices in a weighted graph (by trial-and-error methods only)
- explain the meaning of the terms: Eulerian graph, Eulerian trail, semi-Eulerian graph, semi-Eulerian trail and the conditions for their existence, and use these concepts to investigate and solve practical problems; for example, the Königsberg Bridge problem, planning a garbage bin collection route
- explain the meaning of the terms: Hamiltonian graph and semi-Hamiltonian graph, and use these concepts to investigate and solve practical problems; for example, planning a sight-seeing tourist route around a city, the travelling-salesman problem (by trial-and-error methods only)

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 10-12.

Unit 4: Mathematical Applications

Value: 1.0

Unit 4a: Mathematical Applications

Value: 0.5

Unit 4b: Mathematical Applications

Value: 0.5

Unit Description

This unit has three topics: 'Time series analysis'; 'Loans, investments and annuities' and 'Networks and decision mathematics'.

'Time series analysis' continues students' study of statistics by introducing them to the concepts and techniques of time series analysis. The content is to be taught within the framework of the statistical investigation process.

'Loans, investments and annuities' aims to provide students with sufficient knowledge of financial mathematics to solve practical problems associated with taking out or refinancing a mortgage and making investments.

'Networks and decision mathematics' uses networks to model and aid decision making in practical situations.

Classroom access to the technology necessary to support the graphical, computational and statistical aspects of this unit is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in time series analysis; loans, investments and annuities; and networks and decision mathematics
- apply reasoning skills and solve practical problems in time series analysis; loans, investments and annuities; and networks and decision mathematics
- implement the statistical investigation process in contexts requiring the analysis of time series data
- communicate their arguments and strategies, when solving mathematical and statistical problems, using appropriate mathematical or statistical language
- interpret mathematical and statistical information, and ascertain the reasonableness of their solutions to problems and their answers to statistical questions
- choose and use technology appropriately and efficiently.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Time series analysis

Describing and interpreting patterns in time series data:

- construct time series plots
- describe time series plots by identifying features such as trend (long term direction), seasonality (systematic, calendar-related movements), and irregular fluctuations (unsystematic, short term fluctuations), and recognise when there are outliers; for example, one-off unanticipated events

Analysing time series data:

- smooth time series data by using a simple moving average, including the use of spreadsheets to implement this process
- calculate seasonal indices by using the average percentage method
- deseasonalise a time series by using a seasonal index, including the use of spreadsheets to implement this process
- fit a least-squares line to model long-term trends in time series data

The data investigation process:

- implement the statistical investigation process to answer questions that involve the analysis of time series data

Topic 2: Loans, investments and annuities

Compound interest loans and investments:

- use a recurrence relation to model a compound interest loan or investment, and investigate (numerically or graphically) the effect of the interest rate and the number of compounding periods on the future value of the loan or investment
- calculate the effective annual rate of interest and use the results to compare investment returns and cost of loans when interest is paid or charged daily, monthly, quarterly or six-monthly
- with the aid of a calculator or computer-based financial software, solve problems involving compound interest loans or investments; for example, determining the future value of a loan, the number of compounding periods for an investment to exceed a given value, the interest rate needed for an investment to exceed a given value

Reducing balance loans (compound interest loans with periodic repayments):

- use a recurrence relation to model a reducing balance loan and investigate (numerically or graphically) the effect of the interest rate and repayment amount on the time taken to repay the loan
- with the aid of a financial calculator or computer-based financial software, solve problems involving reducing balance loans; for example, determining the monthly repayments required to pay off a housing loan

Annuities and perpetuities (compound interest investments with periodic payments made from the investment):

- use a recurrence relation to model an annuity, and investigate (numerically or graphically) the effect of the amount invested, the interest rate, and the payment amount on the duration of the annuity
- with the aid of a financial calculator or computer-based financial software, solve problems involving annuities (including perpetuities as a special case); for example, determining the amount to be invested in an annuity to provide a regular monthly income of a certain amount.

Topic 3: Networks and decision mathematics

Trees and minimum connector problems:

- explain the meaning of the terms tree and spanning tree and identify practical examples
- identify a minimum spanning tree in a weighted connected graph either by inspection or by using Prim's algorithm
- use minimal spanning trees to solve minimal connector problems; for example, minimising the length of cable needed to provide power from a single power station to substations in several towns

Project planning and scheduling using critical path analysis (CPA):

- construct a network to represent the durations and interdependencies of activities that must be completed during the project; for example, preparing a meal
- use forward and backward scanning to determine the earliest starting time (EST) and latest starting times (LST) for each activity in the project
- use ESTs and LSTs to locate the critical path(s) for the project
- use the critical path to determine the minimum time for a project to be completed
- calculate float times for non-critical activities

Flow networks

- solve small-scale network flow problems including the use of the 'maximum-flow minimum-cut' theorem; for example, determining the maximum volume of oil that can flow through a network of pipes from an oil storage tank (the source) to a terminal (the sink).

Assignment problems

- use a bipartite graph and/or its tabular or matrix form to represent an assignment/ allocation problem; for example, assigning four swimmers to the four places in a medley relay team to maximise the team's chances of winning
- determine the optimum assignment(s), by inspection for small-scale problems, or by use of the Hungarian algorithm for larger problems

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 10-12.

Appendix A – Implementation Guidelines

Available course patterns

A standard 1.0 value unit is delivered over at least 55 hours. To be awarded a course, students must complete at least the minimum units over the whole minor, major, major/minor or double major course.

| Course | Number of standard units to meet course requirements |
|--------|--|
| Minor | Minimum of 2 units |
| Major | Minimum of 3.5 units |

Units in this course can be delivered in any order.

Prerequisites for the course or units within the course

Nil.

Arrangements for students continuing study in this course

Students who studied the previous course may undertake any units in this course provided there is no duplication of content.

Duplication of Content Rules

Students cannot be given credit towards the requirements for a Senior Secondary Certificate for a unit that significantly duplicates content in a unit studied in another course. The responsibility for preventing undesirable overlap of content studied by a student rests with the principal and the teacher delivering the course. Students will only be given credit for covering the content once.

Relationship to other courses

Students may study this course concurrently with the Mathematical Methods course (integrating Australian Curriculum), Specialist Mathematics course (integrating Australian Curriculum) or BSSS Specialist Methods course (noting the co-requisites for courses).

When units from Mathematical Applications (integrating Australian Curriculum) are combined with any units from BSSS Specialist Methods, Specialist Mathematics (integrating Australian Curriculum) or Mathematical Methods (integrating Australian Curriculum) they form a course in Further Mathematics.

Refer to the Further Mathematics integrated course for details.

NOTE: When units from BSSS Specialist Methods, Specialist Mathematics (integrating Australian Curriculum) or Mathematical Methods (integrating Australian Curriculum) are combined they form a Mathematical Methods courses (integrating Australian Curriculum) or Specialist Mathematics (integrating Australian Curriculum) course based on units studied (no Mathematical Applications units integrating Australian Curriculum can be included).

Guidelines for Delivery

Program of Learning

A program of learning is what a school provides to implement the course for a subject. This meets the requirements for context, scope and sequence set out in the Board endorsed course. Students follow programs of learning in a college as part of their senior secondary studies. The detail, design and layout of a program of learning are a college decision.

The program of learning must be documented to show the planned learning activities and experiences that meet the needs of particular groups of students, taking into account their interests, prior knowledge, abilities and backgrounds. The program of learning is a record of the learning experiences that enable students to achieve the knowledge, understanding and skills of the content descriptions. There is no requirement to submit a program of learning to the OBSSS for approval. The Principal will need to sign off at the end of Year 12 that courses have been delivered as accredited.

Content Descriptions

Are all content descriptions of equal importance? No. It depends on the focus of study. Teachers can customise their program of learning to meet their own students' needs, adding additional content descriptions if desired or emphasising some over others. A teacher must balance student needs with their responsibility to teach all content descriptions. It is mandatory that teachers address all content descriptions and that students engage with all content descriptions.

Half standard 0.5 units

Half standard units appear on the course adoption form but are not explicitly documented in courses. It is at the discretion of the college principal to split a standard 1.0 unit into two half standard 0.5 units. Colleges are required to adopt the half standard 0.5 units. However, colleges are not required to submit explicit documentation outlining their half standard 0.5 units to the BSSS. Colleges must assess students using the half standard 0.5 assessment task weightings outlined in the framework. It is the responsibility of the college principal to ensure that all content is delivered in units approved by the Board.

Moderation

Moderation is a system designed and implemented to:

- provide comparability in the system of school-based assessment
- form the basis for valid and reliable assessment in senior secondary schools
- involve the ACT Board of Senior Secondary Studies and colleges in cooperation and partnership
- maintain the quality of school-based assessment and the credibility, validity and acceptability of Board certificates.

Moderation commences within individual colleges. Teachers develop assessment programs and instruments, apply assessment criteria, and allocate Unit Grades, according to the relevant Course Framework. Teachers within course teaching groups conduct consensus discussions to moderate marking or grading of individual assessment instruments and unit grade decisions.

The Moderation Model

Moderation within the ACT encompasses structured, consensus-based peer review of Unit Grades for all accredited courses over two Moderation Days. In addition to Moderation Days, there is statistical moderation of course scores, including small group procedures, for T courses.

Moderation by Structured, Consensus-based Peer Review

Consensus-based peer review involves the review of student work against system wide criteria and standards and the validation of Unit Grades. This is done by matching student performance with the criteria and standards outlined in the Achievement Standards, as stated in the Framework. Advice is then given to colleges to assist teachers with, or confirm, their judgments. In addition, feedback is given on the construction of assessment instruments.

Preparation for Structured, Consensus-based Peer Review

Each year, teachers of Year 11 are asked to retain originals or copies of student work completed in Semester 2. Similarly, teachers of a Year 12 class should retain originals or copies of student work completed in Semester 1. Assessment and other documentation required by the Office of the Board of Senior Secondary Studies should also be kept. Year 11 work from Semester 2 of the previous year is presented for review at Moderation Day 1 in March, and Year 12 work from Semester 1 is presented for review at Moderation Day 2 in August.

In the lead up to Moderation Day, a College Course Presentation (comprised of a document folder and a set of student portfolios) is prepared for each A, T and M course/units offered by the school and is sent into the Office of the Board of Senior Secondary Studies.

The College Course Presentation

The package of materials (College Course Presentation) presented by a college for review on Moderation Days in each course area will comprise the following:

- a folder containing supporting documentation as requested by the Office of the Board through memoranda to colleges, including marking schemes and rubrics for each assessment item
- a set of student portfolios containing marked and/or graded written and non-written assessment responses and completed criteria and standards feedback forms. Evidence of all assessment responses on which the Unit Grade decision has been made is to be included in the student review portfolios.

Specific requirements for subject areas and types of evidence to be presented for each Moderation Day will be outlined by the Board Secretariat through the *Requirements for Moderation Memoranda* and Information Papers.

Visual evidence for judgements made about practical performances

It is a requirement that schools' judgements of standards to practical performances (A/T/M) be supported by visual evidence (still photos or video).

The photographic evidence submitted must be drawn from practical skills performed as part of the assessment process.

Teachers should consult the BSSS website for current information regarding all moderation requirements including subject specific and photographic evidence.

Appendix B – Course Developers

| Name | College |
|-----------------|----------------------------------|
| Jacob Woolley | Canberra College |
| Gary Pocock | Canberra Institute of Technology |
| Marion McIntosh | Melba Copland Secondary School |
| Wayne Semmens | Melba Copland Secondary School |
| Jennifer Missen | Merici College |
| Nicole Burg | Narrabundah College |
| Rebecca Guinane | Narrabundah College |
| Andrew Trost | Narrabundah College |

Appendix C – Common Curriculum Elements

Common curriculum elements assist in the development of high-quality assessment tasks by encouraging breadth and depth and discrimination in levels of achievement.

| Organisers | Elements | Examples |
|----------------------------------|------------------|---|
| create, compose and apply | apply | ideas and procedures in unfamiliar situations, content and processes in non-routine settings |
| | compose | oral, written and multimodal texts, music, visual images, responses to complex topics, new outcomes |
| | represent | images, symbols or signs |
| | create | creative thinking to identify areas for change, growth and innovation, recognise opportunities, experiment to achieve innovative solutions, construct objects, imagine alternatives |
| | manipulate | images, text, data, points of view |
| analyse, synthesise and evaluate | justify | arguments, points of view, phenomena, choices |
| | hypothesise | statement/theory that can be tested by data |
| | extrapolate | trends, cause/effect, impact of a decision |
| | predict | data, trends, inferences |
| | evaluate | text, images, points of view, solutions, phenomenon, graphics |
| | test | validity of assumptions, ideas, procedures, strategies |
| | argue | trends, cause/effect, strengths and weaknesses |
| | reflect | on strengths and weaknesses |
| | synthesise | data and knowledge, points of view from several sources |
| | analyse | text, images, graphs, data, points of view |
| | examine | data, visual images, arguments, points of view |
| | investigate | issues, problems |
| organise, sequence and explain | sequence | text, data, relationships, arguments, patterns |
| | visualise | trends, futures, patterns, cause and effect |
| | compare/contrast | data, visual images, arguments, points of view |
| | discuss | issues, data, relationships, choices/options |
| | interpret | symbols, text, images, graphs |
| | explain | explicit/implicit assumptions, bias, themes/arguments, cause/effect, strengths/weaknesses |
| | translate | data, visual images, arguments, points of view |
| | assess | probabilities, choices/options |
| | select | main points, words, ideas in text |
| identify, summarise and plan | reproduce | information, data, words, images, graphics |
| | respond | data, visual images, arguments, points of view |
| | relate | events, processes, situations |
| | demonstrate | probabilities, choices/options |
| | describe | data, visual images, arguments, points of view |
| | plan | strategies, ideas in text, arguments |
| | classify | information, data, words, images |
| | identify | spatial relationships, patterns, interrelationships |
| | summarise | main points, words, ideas in text, review, draft and edit |

Appendix D – Glossary of Verbs

| Verbs | Definition |
|--------------------|--|
| Analyse | Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences |
| Apply | Use, utilise or employ in a particular situation |
| Argue | Give reasons for or against something |
| Assess | Make a judgement about the value of |
| Classify | Arrange into named categories in order to sort, group or identify |
| Compare | Estimate, measure or note how things are similar or dissimilar |
| Compose | The activity that occurs when students produce written, spoken, or visual texts |
| Contrast | Compare in such a way as to emphasise differences |
| Create | Bring into existence, to originate |
| Critically analyse | Analysis that engages with criticism and existing debate on the issue |
| Demonstrate | Give a practical exhibition an explanation |
| Describe | Give an account of characteristics or features |
| Discuss | Talk or write about a topic, taking into account different issues or ideas |
| Evaluate | Examine and judge the merit or significance of something |
| Examine | Determine the nature or condition of |
| Explain | Provide additional information that demonstrates understanding of reasoning and /or application |
| Extrapolate | Infer from what is known |
| Hypothesise | Put forward a supposition or conjecture to account for certain facts and used as a basis for further investigation by which it may be proved or disproved |
| Identify | Recognise and name |
| Interpret | Draw meaning from |
| Investigate | Planning, inquiry into and drawing conclusions about |
| Justify | Show how argument or conclusion is right or reasonable |
| Manipulate | Adapt or change |
| Plan | Strategize, develop a series of steps, processes |
| Predict | Suggest what might happen in the future or as a consequence of something |
| Reflect | The thought process by which students develop an understanding and appreciation of their own learning. This process draws on both cognitive and affective experience |
| Relate | Tell or report about happenings, events or circumstances |
| Represent | Use words, images, symbols or signs to convey meaning |
| Reproduce | Copy or make close imitation |
| Respond | React to a person or text |
| Select | Choose in preference to another or others |
| Sequence | Arrange in order |
| Summarise | Give a brief statement of the main points |
| Synthesise | Combine elements (information/ideas/components) into a coherent whole |
| Test | Examine qualities or abilities |
| Translate | Express in another language or form, or in simpler terms |
| Visualise | The ability to decode, interpret, create, question, challenge and evaluate texts that communicate with visual images as well as, or rather than, words |

Appendix E – Glossary for ACT Senior Secondary Curriculum

Courses will detail what teachers are expected to teach and students are expected to learn for year 11 and 12. They will describe the knowledge, understanding and skills that students will be expected to develop for each learning area across the years of schooling.

Learning areas are broad areas of the curriculum, including English, mathematics, science, the arts, languages, health and physical education.

A **subject** is a discrete area of study that is part of a learning area. There may be one or more subjects in a single learning area.

Frameworks are system documents for Years 11 and 12 which provide the basis for the development and accreditation of any course within a designated learning area. In addition, frameworks provide a common basis for assessment, moderation and reporting of student outcomes in courses based on the framework.

The **course** sets out the requirements for the implementation of a subject. Key elements of a course include the rationale, goals, content descriptions, assessment, and achievement standards as designated by the framework.

BSSS courses will be organised into units. A unit is a distinct focus of study within a course. A standard 1.0 unit is delivered for a minimum of 55 hours generally over one semester.

Core units are foundational units that provide students with the breadth of the subject.

Additional units are avenues of learning that cannot be provided for within the four core 1.0 standard units by an adjustment to the program of learning.

An **Independent Study unit** is a pedagogical approach that empowers students to make decisions about their own learning. Independent Study units can be proposed by a student and negotiated with their teacher but must meet the specific unit goals and content descriptions as they appear in the course.

An **elective** is a lens for demonstrating the content descriptions within a standard 1.0 or half standard 0.5 unit.

A **lens** is a particular focus or viewpoint within a broader study.

Content descriptions refer to the subject-based knowledge, understanding and skills to be taught and learned.

A **program of learning** is what a college develops to implement the course for a subject and to ensure that the content descriptions are taught and learned.

Achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following completion of study of senior secondary course content for units in a subject.

ACT senior secondary system **curriculum** comprises all BSSS approved courses of study.

Appendix F – Mathematical Applications Glossary

Financial Mathematics

Annuity

An **annuity** is a compound interest investment from which payments are made on a regular basis for a fixed period of time. At the end of this time the investment has no residual value.

Book value

The **book value** is the value of an asset recorded on a balance sheet. The book value is based on the original cost of the asset less depreciation.

For example, if the original cost of a printer is \$500 and its value depreciates by \$100 over the next year, then its book value at the end of the year is \$400.

There are three commonly used methods for calculating yearly depreciation in the value of an asset, namely, **reducing balance depreciation**, **flat rate depreciation** or **unit cost depreciation**.

CPI

The **Consumer Price Index** (CPI) is a measure of changes, over time, in retail prices of a constant basket of goods and services representative of consumption expenditure by resident households in Australian metropolitan areas.

Effective annual rate of interest

The **effective annual rate of interest** $i_{\text{effective}}$ is used to compare the interest paid on loans (or investments) with the same nominal annual interest rate i but with different compounding periods (daily, monthly, quarterly, annually, other)

If the number of compounding periods per annum is n , then $i_{\text{effective}} = (1 + \frac{i}{n})^n - 1$

For example if the quoted annual interest rate for a loan is 9%, but interest is charged monthly, then

the effective annual interest rate charged is $i_{\text{effective}} = 1 + \frac{0.09}{12}^{12} - 1 = 0.9416\dots$, or around 9.4%.

Diminishing value depreciation see Reducing balance depreciation

Flat rate depreciation

In flat rate or straight-line depreciation the value of an asset is depreciated by a fixed amount each year. Usually this amount is specified as a fixed percentage of the original cost.

GST

The **GST** (Goods and Services Tax) is a broad sales tax of 10% on most goods and services transactions in Australia.

Straight-line depreciation

See: flat rate depreciation

Compound interest

The interest earned by investing a sum of money (the principal) is compound interest if each successive interest payment is added to the principal for the purpose of calculating the next interest payment.

For example, if the principal P earns compound interest at the rate of $i\%$ per period, then after n periods the total amount accrued is $P\left(1 + \frac{i}{100}\right)^n$. When plotted on a graph, the total amount accrued is seen to grow exponentially.

Perpetuity

A **perpetuity** is a compound interest investment from which payments are made on a regular basis in perpetuity (forever). This is possible because the payments made at the end of each period exactly equal the interest earned during that period.

Price to earnings ratio (of a share)

The price to earnings ratio of a share (P/E ratio) is defined as :

$$P/E \text{ ratio} = \frac{\text{Market price per share}}{\text{Annual earnings per share}}$$

Reducing balance depreciation

In **reducing balance depreciation** the value of an asset is depreciated by a fixed percentage of its value each year.

Reducing balance depreciation is sometimes called diminishing value depreciation.

Reducing balance loan

A reducing balance loan is a compound interest loan where the loan is repaid by making regular payments and the interest paid is calculated on the amount still owing (the reducing balance of the loan) after each payment is made.

Simple interest

Simple interest is the interest accumulated when the interest payment in each period is a fixed fraction of the principal. For example, if the principal P earns simple interest at the rate of $i\%$ per period, then after n periods the accumulated simple interest is $nP \frac{i}{100}$

When plotted on a graph, the total amount accrued is seen to grow linearly.

Unit cost depreciation

In unit cost depreciation, the value of an asset is depreciated by an amount related to the number of units produced by the asset during the year.

Geometry and trigonometry

Angle of elevation

The angle a line makes above a plane.

Angle of depression

The angle a line makes below a plane.

Area of a triangle

The general rule for determining the area of a triangle is: $area = \frac{1}{2} base \times height$

Bearings (compass and true)

A **bearing** is the direction of a fixed point, or the path of an object, from the point of observation.

Compass bearings are specified as angles either side of north or south. For example a compass bearing of N50°E is found by facing north and moving through an angle of 50° to the East.

True (or three figure) bearings are measured in degrees from the north line. Three figures are used to specify the direction. Thus the direction of north is specified as 000°, east is specified as 090°, south is specified as 180° and north-west is specified as 315°.

Cosine rule

For a triangle of side lengths a , b and c and angles A , B and C , the **cosine rule** states that

$$c^2 = a^2 + b^2 - 2ab \cos C$$

Heron's rule

Heron's rule is a rule for determining the area of a triangle given the lengths of its sides.

The area A of a triangle of side lengths a , b and c is given by $A = \sqrt{s(s-a)(s-b)(s-c)}$ where

$$s = \frac{1}{2}(a + b + c).$$

Similar figures

Two geometric figures are similar if they are of the same shape but not necessarily of the same size.

Sine rule

For a triangle of side lengths a , b and c and angles A , B and C , the **sine rule** states that

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Triangulation

The process of determining the location of a point by measuring angles to it from known points at either end of a fixed baseline, rather than measuring distances to the point directly. The point can then be fixed as the third point of a triangle with one known side and two known angles.

Scale factor

A **scale factor** is a number that scales, or multiplies, some quantity. In the equation $y = kx$, k is the scale factor for x .

If two or more figures are similar, their sizes can be compared. The scale factor is the ratio of the length of one side on one figure to the length of the corresponding side on the other figure. It is a measure of magnification, the change of size.

Graphs & networks

Adjacent (graph)

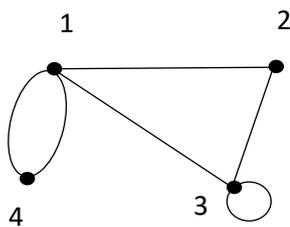
see graph

Adjacency matrix

An **adjacency matrix** for a non-directed graph with n vertices is a $n \times n$ matrix in which the entry in row i and column j is the number of edges joining the vertices i and j . In an adjacency matrix, a **loop** is counted as 1 edge.

Example:

Non-directed graph



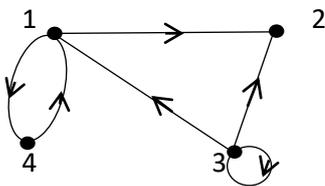
Adjacency matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 1 | 1 | 1 |
| 2 | 1 | 0 | 1 | 0 |
| 3 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 |

For a directed graph the entry in row i and column j is the number of directed edges (arcs) joining the vertex i and j in the direction i to j .

Example:

Directed graph



Adjacency matrix

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1 | 2 | 1 | 0 | 1 |
| 2 | 0 | 0 | 0 | 0 |
| 3 | 1 | 1 | 1 | 0 |
| 4 | 1 | 0 | 0 | 0 |

Algorithm

An **algorithm** is a precisely defined routine procedure that can be applied and systematically followed through to a conclusion. An example is **Prim's algorithm** for determining a **minimum spanning tree** in a network.

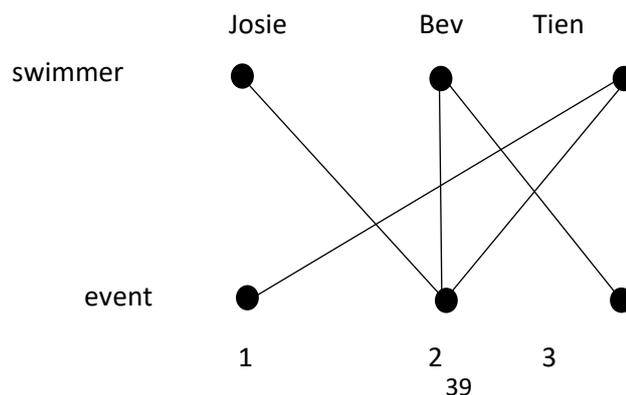
Arc

see directed graph

Bipartite Graph

A bipartite graph is a graph whose set of vertices can be split into two distinct groups in such a way that each edge of the graph joins a vertex in the first group to a vertex in the second group.

Example:



Bridge

See connected graph

Closed path

See path

Closed trail

See trail

Closed walk

See walk

Complete graph

A **complete graph** is a **simple graph** in which every vertex is joined to every other vertex by an edge.

The complete graph with n vertices is denoted K_n .

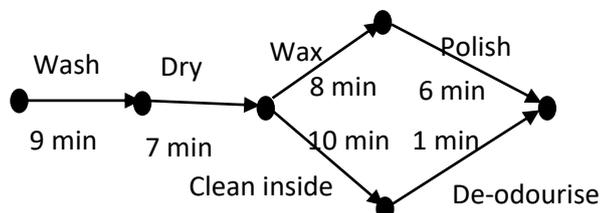
Connected graph

A graph is **connected** if there is a path between each pair of vertices. A **bridge** is an edge in a connected graph that, if removed, leaves a graph disconnected.

Critical path analysis (CPA)

A project often involves many related activities some of which cannot be started until one or more earlier tasks have been completed. One way of scheduling such activities that takes this into account is to construct a network diagram.

The network diagram below can be used to schedule the activities of two or more individuals involved in cleaning and polishing a car. The completion times for each activity are also shown.



Critical path analysis is a method for determining the longest path (the **critical path**) in such a network and hence the minimum time in which the project can be completed. There may be more than one critical path in the network. In this project the critical path is 'Wash-Dry-Wax-Polish' with a total completion time of 30 minutes.

The **earliest starting time (EST)** of an activity 'Polish' is 24 minutes because activities 'Wash', 'Dry' and 'Wax' must be completed first. The process of systematically determining earliest starting times is called **forward scanning**.

The shortest time that the project can be completed is 30 minutes. Thus, the **latest starting time (LST)** for the activity 'De-odourise' is 29 minutes. The process of systematically determining latest starting times is called **backward scanning**.

Float or slack

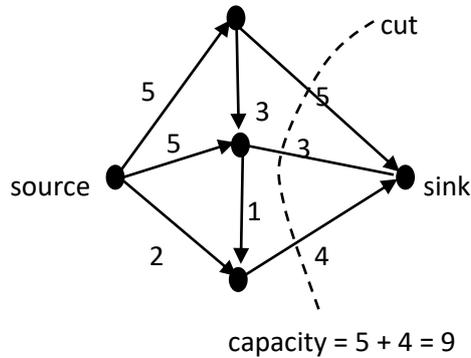
Is the amount of time that a task in a project network can be delayed without causing a delay to subsequent tasks. For example, the activity 'De-odourise' is said to have a **float** of 3 minutes because its earliest EST (26 minutes) is three minutes before its LST (29 minutes). As a result this activity can be started at any time between 26 and 29 minutes after the project started. All activities on a critical path have zero floats.

Cut (in a flow network)

In a flow network, a **cut** is a partition of the vertices of a graph into two separate groups with the **source** in one group and the **sink** in the other.

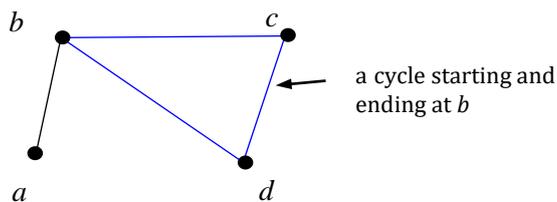
The capacity of the **cut** is the sum of the capacities of the cut edges directed from source to sink. Cut edges directed from sink to source are ignored.

Example:



Cycle

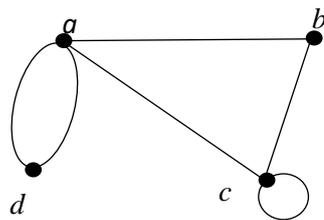
A **cycle** is a closed walk which begins and starts at the same vertex and which has no repeated edges or vertices except the first. If a, b, c and d are the vertices of a graph, the closed walk $bcd b$ that starts and ends at vertex b (shown in blue) is an example of a cycle.



Degree of a vertex (graph)

In a graph, the **degree of a vertex** is the number of edges incident with the vertex, with loops counted twice. It is denoted $\text{deg } v$.

In the graph below, $\text{deg } a = 4$, $\text{deg } b = 2$, $\text{deg } c = 4$ and $\text{deg } d = 2$.

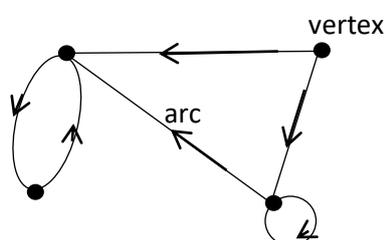


Digraph

See directed graph

Directed graph

A **directed graph** is a diagram comprising points, called vertices, joined by directed lines called **arcs**. The directed graphs are commonly called **digraphs**.



Earliest starting time (EST)

See Critical Path Analysis

Edge

See graph

Euler's formula

For a connected planar graph, **Euler's rule** states that

$$v + f - e = 2$$

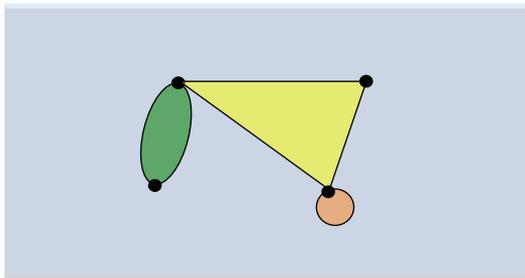
where v is the number vertices, e is the number of edges and f is the number of faces.

Eulerian graph

A connected graph is **Eulerian** if it has a closed trail (starts and ends at the same vertex), that is, includes every edge and once only; such a trail is called an **Eulerian trail**. An Eulerian trial may include repeated vertices. A connected graph is **semi-Eularian** if there is an open trail that includes every edge once only.

Face

The faces of a planar graph are the regions bounded by the edges including the outer infinitely large region. The planar graph shown has four faces.



Float time

See Critical Path Analysis

Flow network

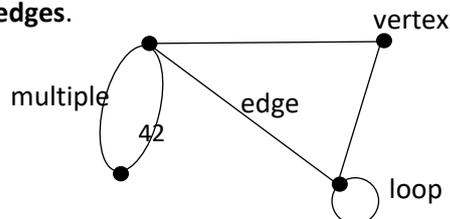
A **flow network** is a directed graph where each edge has a capacity (e.g. 100 cars per hour, 800 litres per minute, etc) and each edge receives a flow. The amount of flow on an edge cannot exceed the capacity of the edge. A flow must satisfy the restriction that the amount of flow into a node equals the amount of flow out of it, except when it is a **source**, which has more outgoing flow, or a **sink**, which has more incoming flow. A flow network can be used to model traffic in a road system, fluids in pipes, currents in an electrical circuit, or any situation in which something travels through a network of nodes.

Food web

A **food web** (or food chain) depicts feeding connections (who eats whom) in an ecological community.

Graph

A **graph** is a diagram that consists of a set of points, called **vertices** that are joined by a set of lines called **edges**. Each edge joins two vertices. A **loop** is an edge in a graph that joins a **vertex** in a **graph** to itself. Two vertices are **adjacent** if they are joined by an edge. Two or more edges which connect the same vertices are called **multiple edges**.



Hamiltonian cycle

A **Hamiltonian cycle** is a **cycle** that includes each **vertex** in a **graph** (except the first), once only.

Hamiltonian path

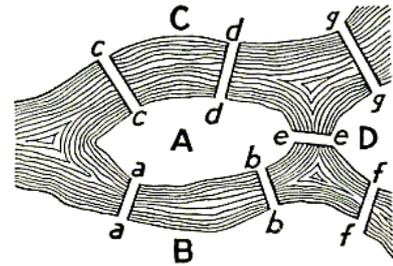
A **Hamiltonian path** is a path that includes every **vertex** in a **graph** once only. A Hamiltonian path that begins and ends at the same vertex is a Hamiltonian cycle.

Hungarian algorithm

The Hungarian algorithm is used to solve assignment (allocation) problems.

Königsberg bridge problem

The Königsberg bridge problem asks: Can the seven bridges of the city of Königsberg all be traversed in a single trip that starts and finishes at the same place?



Latest starting time (LST)

See Critical Path Analysis

Length (of a walk)

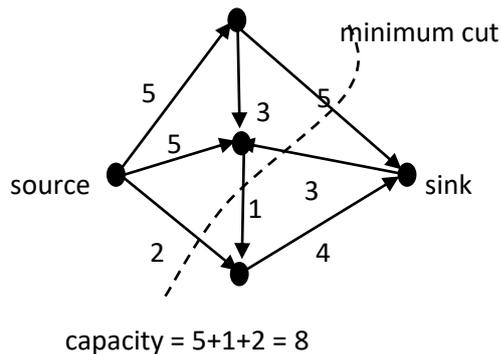
The **length** of a walk is the number of edges it includes.

Minimum cut-maximum flow theorem

The **maximum flow–minimum cut theorem** states that in a flow network, the maximum flow from the **source** to the **sink** is equal to the capacity of the **minimum cut**.

In everyday language, the minimum cut involves identifying the ‘bottle-neck’ in the system.

Example:



Minimum spanning tree

For a given connected **weighted graph**, the **minimum spanning tree** is the **spanning tree** of minimum length.

Multiple edges

see graph

Network

The word network is frequently used in everyday life, e.g. television network, rail network, etc. Weighted graphs or digraphs can often be used to model such networks.

Open path

See path

Open walk

See walk

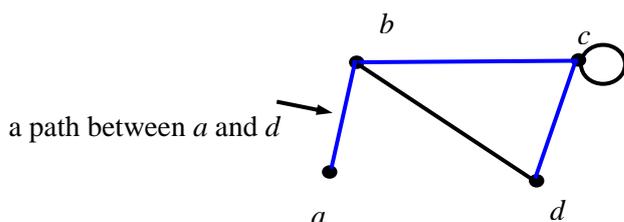
Open trail

See trail

Path (in a graph)

A **path** in a graph is a walk in which all of the edges and all the vertices are different. A path that starts and finishes at different vertices is said to be open, while a path that starts and finishes at the same vertex is said to be closed. A cycle is a closed path.

If a and d are the vertices of a graph, a walk from a to d along the edges coloured blue is a path. Depending on the graph, there may be multiple paths between the same two vertices, as is the case here.



Planar graph

A **planar graph** is a graph that can be drawn in the plane. A planar graph can always be drawn so that no two edges cross.

Prim's algorithm

An **algorithm** for determining a **minimum spanning tree** in a connected weighted graph.

Round-robin sporting competition

A single round robin sporting competition is a competition in which each competitor plays each other competitor once only.

Semi-Eulerian graph

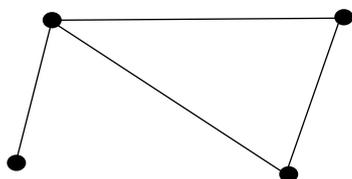
See Eulerian graph

Semi-Hamiltonian graph

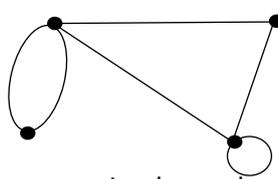
See Hamiltonian graph

Simple graph

A simple graph has no loops or multiple edges.



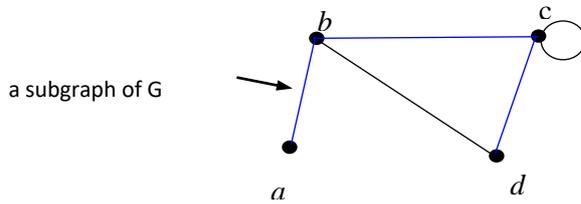
simple graph



non-simple graph

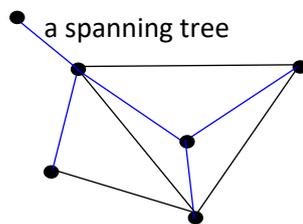
Subgraph

When the vertices and edges of a graph A (shown in blue) are the vertices and edges of the graph G , graph A is said to be a **subgraph** of graph G .



Spanning tree

A **spanning tree** is a **subgraph** of a **connected graph** that connects all vertices and is also a **tree**.



Trail

A **trail** is a **walk** in which no edge is repeated.

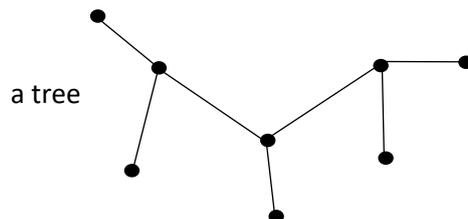
The travelling salesman problem

The travelling salesman problem can be described as follows: Given a list of cities and the distance between each city, find the shortest possible route that visits each city exactly once.

While in simple cases this problem can be solved by systematic identification and testing of possible solutions, no there is no known efficient method for solving this problem.

Tree

A **tree** is a connected graph with no cycles.



Vertex

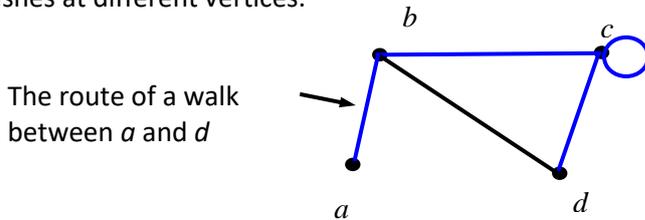
See graph

Walk (in a graph)

A **walk** in a graph is a sequence of vertices such that from each of its vertices there is an edge to the next vertex in the sequence. A walk that starts and finishes at different vertices is said to be an **open walk**. A walk that starts and finishes at the same vertex is said to be a **closed walk**.

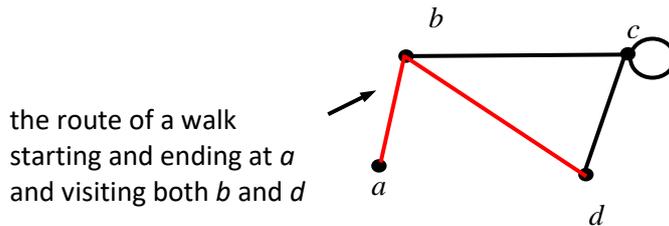
If a, b, c and d are the vertices of a graph with edges ab, bc, cd and bd , then the sequence of edges (ab, bc, cd) constitute a walk. The route followed on this walk is shown in blue on the graph below.

This walk is denoted by the sequence of vertices $abcc$. The walk is open because it begins and finishes at different vertices.



A walk can include repeated vertices (as is the case above) or repeated edges.

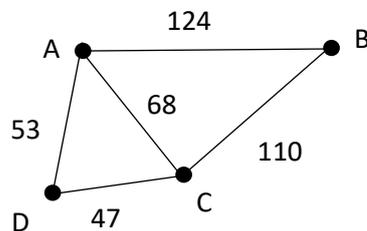
An example of a closed walk with both repeated edges and hence vertices is defined by the sequence of edges (ab, bd, db, ba) and is denoted by the sequence of vertices $abdba$. The route followed is shown in red in the graph below.



Depending on the graph, there may be multiple walks between the same two vertices, as is the case here.

Weighted graph

A weighted graph is a graph in which each edge is labelled with a number used to represent some quantity associated with the edge. For example, if the vertices represent towns, the weights on the edges may represent the distances in kilometres between the towns.



Growth and decay in sequences

Arithmetic sequence

An **arithmetic sequence** is a sequence of numbers such that the difference between any two successive members of the sequence is constant.

For example, the sequence

2, 5, 8, 11, 14, 17, ...

is an arithmetic sequence with first term 2 and common difference 3.

By inspection of the sequence, the rule for the n th term t_n of this sequence is:

$$t_n = 2 + (n - 1)3 = 3n - 1 \quad n \geq 1$$

If t_n is used to denote the n th term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_{n+1} = t_n + 3 \quad n \geq 1$

Break-even point

The **break-even point** is the point at which revenue begins to exceed the cost of production.

First-order linear recurrence relation

A **first-order linear recurrence relation** is defined by the rule: $t_0 = a$, $t_{n+1} = bt_n + c$ for $n \geq 1$

For example, the rule: $t_0 = 10$, $t_n = 5t_{n-1} + 1$ for $n \geq 1$ is a first-order recurrence relation.

The sequence generated by this rule is: 10, 51, 256, ... as shown below.

$$t_1 = 10, t_2 = 5t_1 + 1 = 5 \times 10 + 1 = 51, t_3 = 5t_2 + 1 = 5 \times 51 + 1 = 256, \dots$$

Geometric growth or decay (sequence)

A sequence displays geometric growth or decay when each term is some constant multiple (greater or less than one) of the preceding term. A multiple greater than one corresponds to growth. A multiple less than one corresponds to decay.

For example, the sequence:

1, 2, 4, ... displays geometric growth because each term is double the previous term.

100, 10, 0.1, ... displays geometric decay because each term is one tenth of the previous term.

Geometric growth is an example of exponential growth in discrete situations.

Geometric sequence

A **geometric sequence**, is a sequence of numbers where each term after the first is found by multiplying the previous one by a fixed non-zero number called the **common ratio**. For example, the sequence

2, 6, 18, ...

is a geometric sequence with first term 2 and common ratio 3.

By inspection of the sequence, the rule for the n th term of this sequence is:

$$t_n = 2 \times 3^{n-1} \quad n \geq 1$$

If t_n is used to denote the n th term in the sequence, then a recursion relation that will generate this sequence is: $t_1 = 2$, $t_{n+1} = 3t_n \quad n \geq 1$

Linear growth or decay (sequence)

A sequence displays linear growth or decay when the difference between successive terms is constant. A positive constant difference corresponds to linear growth while a negative constant difference corresponds to decay.

Examples:

The sequence, 1, 4, 7, ... displays linear growth because the difference between successive terms is 3.

The sequence, 100, 90, 80, ... displays linear decay because the difference between successive terms is -10 . By definition, arithmetic sequences display linear growth or decay.

Recursion

See recurrence relation

Recurrence relation

A **recurrence relation** is an equation that recursively defines a sequence; that is, once one or more initial terms are given, each further term of the sequence is defined as a function of the preceding terms.

Sequence

A **sequence** is an ordered list of numbers (or objects).

For example 1, 3, 5, 7 is a sequence of numbers that differs from the sequence 3, 1, 7, 5 as order matters.

A sequence maybe finite, for example, 1, 3, 5, 7 (the sequence of the first four odd numbers), or infinite, for example, 1, 3, 5, ... (the sequence of all odd numbers).

Linear equations (relations) and graphs

Linear equation

A linear equation in one variable x is an equation of the form $ax + b = 0$, e.g. $3x + 1 = 0$

A linear equation in two variables x and y is an equation of the form $ax + by + c = 0$,

e.g. $2x - 3y + 5 = 0$

Linear graph

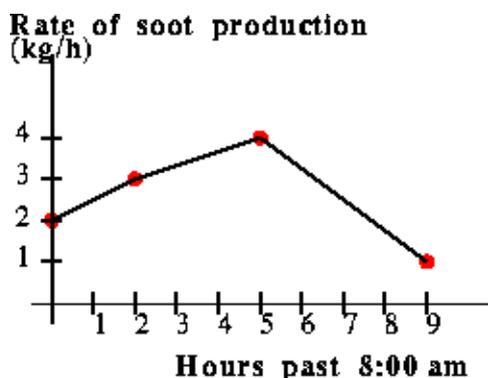
A **linear graph** is a graph of a linear equation with two variables. If the linear equation is written in the form $y = a + bx$, then a represents the y -intercept and b represents the slope (or gradient) of the linear graph.

Piecewise-linear graph

A graph consisting of one or more non overlapping line segments.

Sometimes called a line segment graph.

Example:



Slope (gradient)

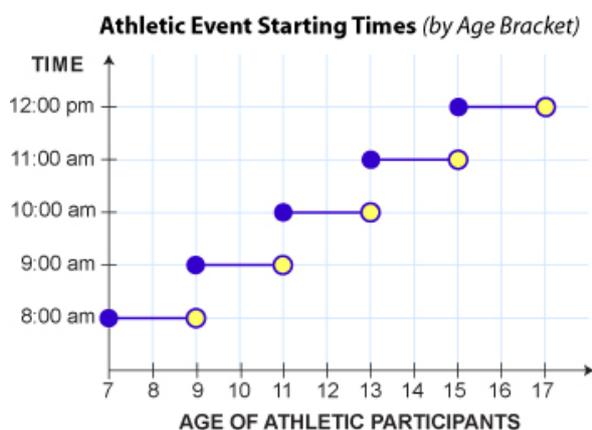
The **slope** or **gradient** of a line describes its steepness, incline, or grade.

Slope is normally described by the ratio of the "rise" divided by the "run" between two points on a line.

See also linear graph.

Step graph

A graph consisting of one or more non-overlapping horizontal line segments that follow a step-like pattern.



Matrices

Addition of matrices

If \mathbf{A} and \mathbf{B} are matrices of the same size (order) and the elements of \mathbf{A} are a_{ij} and the elements of \mathbf{B} are b_{ij} then the elements of $\mathbf{A} + \mathbf{B}$ are $a_{ij} + b_{ij}$

For example if $\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $\mathbf{B} = \begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$ then

$$\mathbf{A} + \mathbf{B} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$$

Elements (Entries) of a matrix

The symbol a_{ij} represents the (i,j) element occurring in the i^{th} row and the j^{th} column.

For example a general 3×2 matrix is:

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \quad \text{where } a_{32} \text{ is the element in the third row and the second column}$$

Identity matrix

A multiplicative **identity matrix** is a square matrix in which all of the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter I .

For example,

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \text{ and } \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{ are both identity matrices.}$$

There is an identity matrix for each size (or order) of a square matrix. When clarity is needed, the order is written with a subscript: I_n

Inverse of a square matrix

The **inverse of a square matrix** \mathbf{A} is written as \mathbf{A}^{-1} and has the property that

$$\mathbf{A}\mathbf{A}^{-1} = \mathbf{A}^{-1}\mathbf{A} = I$$

Not all square matrices have an inverse. A matrix that has an inverse is said to be **invertible**.

Inverse of a 2×2 matrix

The **inverse** of the matrix $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is

$$\mathbf{A}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix} \text{ provided } ad - bc \neq 0$$

Leading diagonal

The leading diagonal of a square matrix is the diagonal that runs from the top left corner to the bottom right corner of the matrix.

Matrix (matrices)

A **matrix** is a rectangular array of elements or entities displayed in rows and columns.

For example,

$$\mathbf{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \text{ are both matrices with six elements.}$$

Matrix **A** is said to be a 3×2 matrix (three rows and two columns) while **B** is said to be a 2×3 matrix (two rows and three columns).

A **square matrix** has the same number of rows and columns.

A **column matrix** (or vector) has only one column.

A **row matrix** (or vector) has only one row.

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix.

For example, forming the product

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 25 \\ 11 & 45 \end{bmatrix}$$

The multiplication is defined by $1 \times 2 + 8 \times 0 + 0 \times 4 = 2$

$$1 \times 1 + 8 \times 3 + 0 \times 4 = 25$$

$$2 \times 2 + 5 \times 0 + 7 \times 1 = 11$$

$$2 \times 1 + 5 \times 3 + 7 \times 4 = 45$$

This is an example of the process of matrix multiplication.

The product **AB** of two matrices **A** and **B** of size $m \times n$ and $p \times q$ respectively is defined if $n = p$.

If $n = p$ the resulting matrix has size $m \times q$.

$$\text{If } \mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} \text{ and } \mathbf{B} = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix} \text{ then}$$

$$\mathbf{AB} = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

Order (of a matrix)

See **size** (of a matrix)

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product

$$10 \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix \mathbf{A} with elements a_{ij} the elements of $k\mathbf{A}$ are ka_{ij} .

Singular matrix

A matrix is singular if $\det \mathbf{A} = 0$. A singular matrix does not have a multiplicative inverse.

Size (of a matrix)

Two matrices are said to have the same **size** (or **order**) if they have the same number of rows and columns. A matrix with m rows and n columns is said to be a $m \times n$ matrix.

For example, the matrices

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \text{ and } \begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$$

have the same size. They are both 2×3 matrices.

Zero matrix

A zero matrix is a matrix if all of its entries are zero. For example:

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \text{ and } \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \text{ are zero matrices.}$$

Statistics

Association

A general term used to describe the relationship between two (or more) variables. The term **association** is often used interchangeably with the term **correlation**. The latter tends to be used when referring to the strength of a linear relationship between two numerical variables.

Average percentage method

In the **average percentage method** for calculating a **seasonal index**, the data for each 'season' are expressed as percentages of the average for the year. The percentages for the corresponding 'seasons' for different years are then averaged using a mean or median to arrive at a seasonal index.

Categorical variable

A **categorical variable** is a variable whose values are categories.

Examples include blood group (A, B, AB or O) or house construction type (brick, concrete, timber, steel, other).

Categories may have numerical labels, eg. the numbers worn by player in a sporting team, but these labels have no numerical significance, they merely serve as labels.

Categorical data

Data associated with a **categorical variable** is called categorical data.

Causation

A relationship between an explanatory and a response variable is said to be causal if the change in the explanatory variable actually causes a change in the response variable. Simply knowing that two variables are associated, no matter how strongly, is not sufficient evidence by itself to conclude that the two variables are causally related.

Possible explanations for an observed association between an explanatory and a response variable include:

- the **explanatory variable** is actually causing a change in the response variable
- there may be causation, but the change may also be caused by one or more uncontrolled variables whose effects cannot be disentangled from the effect of the response variable. This is known as **confounding**.
- there is no causation, the association is explained by at least one other variable that is associated with both the explanatory and the response variable. This is known as a **common response**.
- the **response variable** is actually causing a change in the explanatory variable

Coefficient of determination

In a linear model between two variables, the coefficient of determination (R^2) is the proportion of the total variation that can be explained by the linear relationship existing between the two variables, usually expressed as a percentage. For two variables only, the coefficient of determination is numerically equal to the square of the correlation coefficient (r^2).

Example

A study finds that the correlation between the heart weight and body weight of a sample of mice is $r = 0.765$. The coefficient of determination = $r^2 = 0.765^2 = 0.5852 \dots$ or approximately 59%

From this information, it can be concluded that approximately 59% of the variation in heart weights of these mice can be explained by the variation in their body weights.

Note: The coefficient of determination has a more general and more important meaning in considering relationships between more than two variables, but this is not a school level topic.

Common response

See Causation

Confounding

See Causation

Continuous data

Data associated with a **continuous variable** is called continuous data.

Continuous variable

A **continuous variable** is a **numerical variable** that can take any value that lies within an interval. In practice, the values taken are subject to accuracy of the measurement instrument used to obtain these values.

Examples include height, reaction time and systolic blood pressure.

Correlation

Correlation is a measure of the strength of the linear relationship between two variables. See also **association**.

Correlation coefficient (r)

The correlation coefficient (r) is a measure of the strength of the linear relationship between a pair of variables. The formula for calculating r is given below.

For variables x and y , and computed for n cases, the formula for r is:

$$r = \frac{1}{n-1} \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{s_x s_y}$$

Discrete data

Discrete data is data associated with a discrete variable. Discrete data is sometimes called count data.

Discrete variable

A **discrete variable** is a **numerical variable** that can take only integer values.

Examples include the number of people in a car, the number of decayed teeth in 18 year-old males, etc.

Explanatory variable

When investigating relationships in bivariate data, the **explanatory variable** is the variable used to explain or predict a difference in the **response variable**.

For example, when investigating the relationship between the temperature of a loaf of bread and the time it has spent in a hot oven, *temperature* is the response variable and *time* is the explanatory variable.

Extrapolation

In the context of fitting a linear relationship between two variables, extrapolation occurs when the fitted model is used to make predictions using values of the **explanatory variable** that are outside the range of the original data. Extrapolation is a dangerous process as it can sometimes lead to quite erroneous predictions.

See also interpolation.

Five-number summary

A **five-number summary** is a method of summarising a set of data using the minimum value, the lower or first-quartile (Q_1), the median, the upper or third-quartile (Q_3) and the maximum value. Forms the basis for a boxplot.

Interpolation

In the context of fitting a linear relationship between two variables, interpolation occurs when the fitted model is used to make predictions using values of the **explanatory variable** that lie within the range of the original data.

See also extrapolation.

Irregular variation or noise (time series)

Irregular variation or noise is erratic and short-term variation in a time series that is the product of chance occurrences.

Least-squares line

In fitting a straight-line $y = a + bx$ to the relationship between a response variable y and an explanatory variable x , the **least-squares line** is the line for which the sum of the squared **residuals** is the smallest.

The formula for calculating the slope (b) and the intercept (a) of the least squares line is given below.

For variables x and y computed for n cases, the slope (b) and intercept (a) of the least-squares line are given by:

$$b = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2} \text{ or } b = r \frac{s_y}{s_x} \quad \text{and} \quad a = \bar{y} - b\bar{x}$$

Location

The notion of central or 'typical value' in a sample distribution.

See also **mean**, **median** and **mode**.

Mean

The arithmetic mean of a list of numbers is the sum of the data values divided by the number of values in the list.

In everyday language, the arithmetic mean is commonly called the average.

For example, for the following list of five numbers 2, 3, 3, 6, 8 the mean equals

$$\frac{2 + 3 + 3 + 6 + 8}{5} = \frac{22}{5} = 4.4$$

In more general language, the mean of n observations x_1, x_2, \dots, x_n is $\bar{x} = \frac{\sum x_i}{n}$

Median

The **median** is the value in a set of ordered set of data values that divides the data into two parts of equal size. When there are an odd number of data values, the median is the middle value. When there is an even number of data values, the median is the average of the two central values.

Mode

The **mode** is the most frequently occurring value in a data set.

Moving average

In a time series, a simple moving average is a method used to **smooth** the time series whereby each observation is replaced by a simple average of the observation and its near neighbours. This process reduces the effect of non-typical data and makes the overall trend easier to see.

Note: There are times when it is preferable to use a weighted average rather simple average, but this is not required in the current curriculum.

Outlier

An outlier in a set of data is an observation that appears to be inconsistent with the remainder of that set of data. An outlier is a surprising observation.

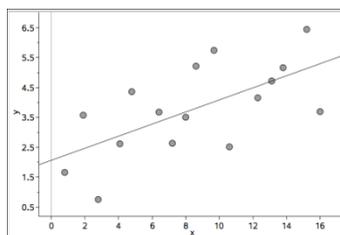
Residual values

The difference between the observed value and the value predicted by a statistical model (e.g., by a least-squares line)

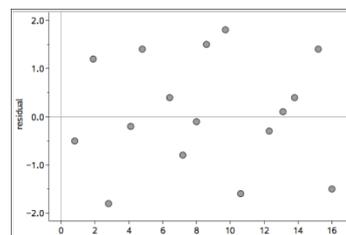
Residual plot

A residual plot is a **scatterplot** with the **residual values** shown on the vertical axis and the **explanatory variable** shown on the horizontal axis. Residual plots are useful in assessing the fit of the statistical model (e.g., by a least-squares line).

When the least-squares line captures the overall relationship between the response variable y and the explanatory variable x , the residual plot will have no clear pattern (be random) see opposite. This is what is hoped for.



scatterplot with least squares line



residual plot

If the least-squares line fails to capture the overall relationship between a response variable and an explanatory variable, a residual plot will reveal a pattern in the residuals. A residual plot will also reveal any outliers that may call into question the use of a least-squares line to describe the relationship. Interpreting patterns in residual plots is a skilled art and is not required in this curriculum.

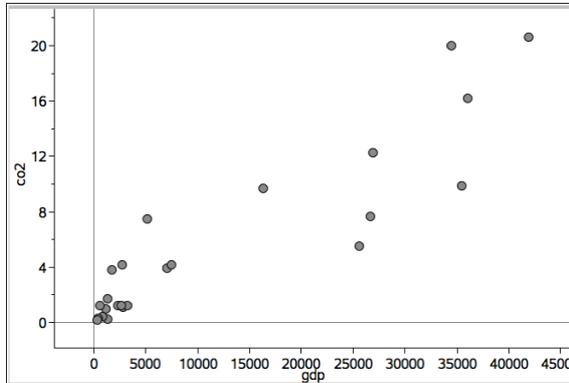
Response variable

See Explanatory variable

Scatterplot

A two-dimensional data plot using Cartesian co-ordinates to display the values of two variables in a bivariate data set.

For example the scatterplot below displays the CO₂ emissions in tonnes per person (*co2*) plotted against Gross Domestic Product per person in \$US (*gdp*) for a sample of 24 countries in 2004. In constructing this scatterplot, *gdp* has been used as the explanatory variable.



Seasonal adjustment (adjusting for seasonality)

A term used to describe a time series from which periodic variations due to seasonal effects have been removed.

See also seasonal index.

Seasonal index

The seasonal index can be used to remove seasonality from data. An index value is attached to each period of the time series within a year. For the seasons of the year (Summer, Autumn, Winter, Spring) there are four separate seasonal indices; for months, there are 12 separate seasonal indices, one for each month, and so on. There are several methods for determining seasonal indices.

Seasonal variation

A regular rise and fall in the time series that recurs each year.

Seasonal variation is measured in terms of a **seasonal index**.

Smoothing (time series) **see moving average**

Standard deviation

The standard deviation is a measure of the variability or spread of a data set. It gives an indication of the degree to which the individual data values are spread around their **mean**.

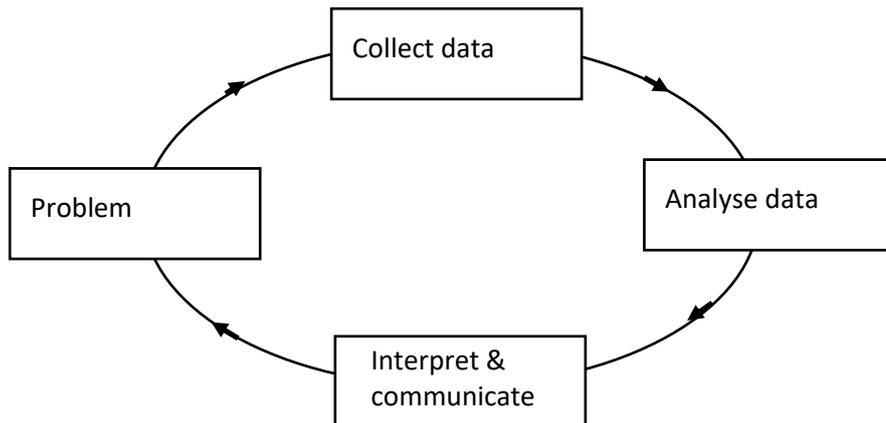
The standard deviation of n observations x_1, x_2, \dots, x_n is

$$s = \sqrt{\frac{\sum (x_i - \bar{x})^2}{n - 1}}$$

Statistical investigation process

The statistical investigation process is a cyclical process that begins with the need to solve a real world problem and aims to reflect the way statisticians work. One description of the statistical investigation process in terms of four steps is as follows.

- Step 1. Clarify the problem and formulate one or more questions that can be answered with data.
- Step 2. Design and implement a plan to collect or obtain appropriate data.
- Step 3. Select and apply appropriate graphical or numerical techniques to analyse the data.
- Step 4. Interpret the results of this analysis and relate the interpretation to the original question; communicate findings in a systematic and concise manner.



Time series

Values of a variable recorded, usually at regular intervals, over a period of time. The observed movement and fluctuations of many such series comprise long-term **trend**, **seasonal variation**, and **irregular variation** or **noise**.

Time series plot

The graph of a **time series** with time plotted on the horizontal axis.

Trend (time series)

Trend is the term used to describe the general direction of a time series (increasing/ decreasing) over a long period of time.

Two-way frequency table

A two-way frequency table is commonly used for displaying the two-way **frequency distribution** that arises when a group of individuals or objects are categorised according to two criteria.

For example, the two-way table below displays the frequency distribution that arises when 27 children are categorised according to *hair type* (straight or curly) and *hair colour* (red, brown, blonde, black).

| Hair colour | Hair type | | Total |
|-------------|-----------|-------|-------|
| | Straight | Curly | |
| red | 1 | 1 | 2 |
| brown | 8 | 4 | 12 |
| blonde | 1 | 3 | 4 |
| black | 7 | 2 | 9 |
| Total | 17 | 10 | 27 |

The row and column totals represent the total number of observations in each row and column and are sometimes called **row sums** or **column sums**.

If the table is 'percentaged' using row sums the resulting percentages are called **row percentages**. If the table is 'percentaged' using column sums the resulting percentages are called **column percentages**.

Appendix G – Course Adoption

Conditions of Adoption

The course and units of this course are consistent with the philosophy and goals of the college and the adopting college has the human and physical resources to implement the course.

Adoption Process

Course adoption must be initiated electronically by an email from the principal or their nominated delegate to bssscertification@ed.act.edu.au. A nominated delegate must CC the principal.

The email will include the **Conditions of Adoption** statement above, and the table below adding the **College** name, and circling the **Classification/s** required.

| | |
|--------------------------|----------------------------------|
| College: | |
| Course Title: | Mathematical Applications |
| Classification/s: | T |
| Accredited from: | 2014 |
| Framework: | Mathematics 2020 |