BSSS

AUSTRALIAN CAPITAL TERRITORY

Specialist Mathematics

T

Cover Art provided by Canberra College student Aidan Giddings

Table of Contents

The ACT Senior Secondary System

The ACT senior secondary system recognises a range of university, vocational or life skills pathways.

The system is based on the premise that teachers are experts in their area: they know their students and community and are thus best placed to develop curriculum and assess students according to their needs and interests. Students have ownership of their learning and are respected as young adults who have a voice.

A defining feature of the system is school-based curriculum and continuous assessment. School-based curriculum provides flexibility for teachers to address students' needs and interests. College teachers have an opportunity to develop courses for implementation across ACT schools. Based on the courses that have been accredited by the BSSS, college teachers are responsible for developing programs of learning. A program of learning is developed by individual colleges to implement the courses and units they are delivering.

Teachers must deliver all content descriptions; however, they do have flexibility to emphasise some content descriptions over others. It is at the discretion of the teacher to select the texts or materials to demonstrate the content descriptions. Teachers can choose to deliver course units in any order and teach additional (not listed) content provided it meets the specific unit goals.

School-based continuous assessment means that students are continually assessed throughout years 11 and 12, with both years contributing equally to senior secondary certification. Teachers and students are positioned to have ownership of senior secondary assessment. The system allows teachers to learn from each other and to refine their judgement and develop expertise.

Senior secondary teachers have the flexibility to assess students in a variety of ways. For example: multimedia presentation, inquiry-based project, test, essay, performance and/or practical demonstration may all have their place. College teachers are responsible for developing assessment instruments with task specific rubrics and providing feedback to students.

The integrity of the ACT Senior Secondary Certificate is upheld by a robust, collaborative and rigorous structured consensus-based peer reviewed moderation process. System moderation involves all Year 11 and 12 teachers from public, non-government and international colleges delivering the ACT Senior Secondary Certificate.

Only students who desire a pathway to university are required to sit a general aptitude test, referred to as the ACT Scaling Test (AST), which moderates student course scores across subjects and colleges. Students are required to use critical and creative thinking skills across a range of disciplines to solve problems. They are also required to interpret a stimulus and write an extended response.

Senior secondary curriculum makes provision for student-centred teaching approaches, integrated and project-based learning inquiry, formative assessment and teacher autonomy. ACT Senior Secondary Curriculum makes provision for diverse learners and students with mild to moderate intellectual disabilities, so that all students can achieve an ACT Senior Secondary Certificate.

The ACT Board of Senior Secondary Studies (BSSS) leads senior secondary education. It is responsible for quality assurance in senior secondary curriculum, assessment and certification. The Board consists of representatives from colleges, universities, industry, parent organisations and unions. The Office of the Board of Senior Secondary Studies (OBSSS) consists of professional and administrative staff who support the Board in achieving its objectives and functions.

ACT Senior Secondary Certificate

Courses of study for the ACT Senior Secondary Certificate:

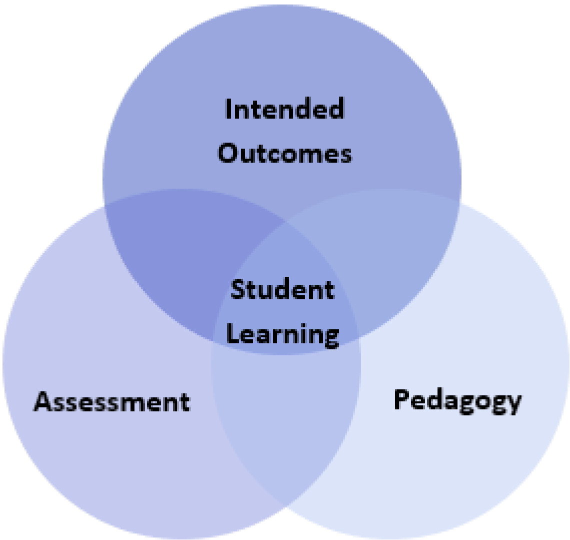
- provide a variety of pathways, to meet different learning needs and encourage students to complete their secondary education
- enable students to develop the essential capabilities for twenty-first century learners
- empower students as active participants in their own learning
- engage students in contemporary issues relevant to their lives
- foster students' intellectual, social and ethical development
- nurture students' wellbeing, and physical and spiritual development
- enable effective and respectful participation in a diverse society.

Each course of study:

- comprises an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum
- is based on a model of learning that integrates intended student outcomes, pedagogy and assessment
- outlines teaching strategies which are grounded in learning principles and encompass quality teaching
- promotes intellectual quality, establish a rich learning environment and generate relevant connections between learning and life experiences
- provides formal assessment and certification of students' achievements.

Underpinning beliefs

- All students are able to learn.
- Learning is a partnership between students and teachers.
- Teachers are responsible for advancing student learning.



Learning Principles

1. Learning builds on existing knowledge, understandings and skills.
   (Prior knowledge)
2. When learning is organised around major concepts, principles and significant real world issues, within and across disciplines, it helps students make connections and build knowledge structures.
   (Deep knowledge and connectedness)
3. Learning is facilitated when students actively monitor their own learning and consciously develop ways of organising and applying knowledge within and across contexts.
   (Metacognition)
4. Learners' sense of self and motivation to learn affects learning.
   (Self-concept)
5. Learning needs to take place in a context of high expectations.
   (High expectations)
6. Learners learn in different ways and at different rates.
   (Individual differences)
7. Different cultural environments, including the use of language, shape learners' understandings and the way they learn.
   (Socio-cultural effects)
8. Learning is a social and collaborative function as well as an individual one.
   (Collaborative learning)
9. Learning is strengthened when learning outcomes and criteria for judging learning are made explicit and when students receive frequent feedback on their progress.
   (Explicit expectations and feedback)

General Capabilities

All courses of study for the ACT Senior Secondary Certificate should enable students to develop essential capabilities for twenty-first century learners. These 'capabilities' comprise an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum.

The capabilities include:

- literacy
- numeracy
- information and communication technology (ICT)
- critical and creative thinking
- personal and social
- ethical behaviour
- intercultural understanding

Courses of study for the ACT Senior Secondary Certificate should be both relevant to the lives of students and incorporate the contemporary issues they face. Hence, courses address the following three priorities. These priorities are:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia's engagement with Asia
- Sustainability

Elaboration of these General Capabilities and priorities is available on the ACARA website at www.australiancurriculum.edu.au.

Literacy in Mathematics

In the senior years these literacy skills and strategies enable students to express, interpret, and communicate complex mathematical information, ideas and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy in Mathematics

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years F to 10. This subject contains topics that will equip students for the ever-increasing demands of the information age.

Information and Communication Technology (ICT) Capability in Mathematics

In the senior years students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. They use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and Creative Thinking in Mathematics

Students compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and Social Capability in Mathematics

In the senior years students develop personal and social competence in Mathematics through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork and decision-making.

The elements of personal and social competence relevant to Mathematics mainly include the application of mathematical skills for their decision-making, life-long learning, citizenship and self-management. In addition, students will work collaboratively in teams and independently as part of their mathematical explorations and investigations.

Ethical Understanding in Mathematics

In the senior years students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural Understanding in Mathematics

Students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person’s environment and language.

Cross-Curriculum Priorities

Aboriginal and Torres Strait Islander Histories and Cultures

The Senior Secondary Mathematics curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

Asia and Australia's Engagement with Asia

There are strong social, cultural and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of mathematics in an Asian context that students engage with Australia's place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

Each of the senior Mathematics subjects provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss this global issue and learn the importance of respecting and valuing a wide range of world perspectives.

Specialist Mathematics T

Rationale

Mathematics is the study of order, relation and pattern. From its origins in counting and measuring it has evolved in highly sophisticated and elegant ways to become the language now used to describe much of the modern world. Statistics is concerned with collecting, analysing, modelling and interpreting data in order to investigate and understand real world phenomena and solve problems in context. Together, mathematics and statistics provide a framework for thinking and a means of communication that is powerful, logical, concise and precise.

Because both mathematics and statistics are widely applicable as models of the world around us, there is ample opportunity for problem solving throughout Specialist Mathematics. There is also a sound logical basis to this subject, and in mastering the subject students will develop logical reasoning skills to a high level.

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical and statistical models more extensively. Topics are developed systematically and lay the foundations for future studies in quantitative subjects in a coherent and structured fashion. Students of Specialist Mathematics will be able to appreciate the true nature of mathematics, its beauty and its functionality.

Specialist Mathematics has been designed to be taken in conjunction with Mathematical Methods or Specialist Methods. The subject contains topics in functions, calculus, probability and statistics that build on and deepen the ideas presented in Mathematical Methods and demonstrate their application in many areas. Vectors, complex numbers and matrices are introduced. Specialist Mathematics is designed for students with a strong interest in mathematics, including those intending to study mathematics, statistics, all sciences and associated fields, economics or engineering at university.

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still applicable and should be inherent in students’ learning of the subject. These strands are Understanding, Fluency, Problem solving and Reasoning and they are both essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency of skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves, freeing up working memory for more complex aspects of problem solving. In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes on an important role and the ability to present the solution of any problem in a logical and clear manner is of paramount importance. The ability to transfer skills learned to solve one class of problems, for example integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject.

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students’ mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit blends algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, in Unit 1 vectors for two-dimensional space are introduced and then in Unit 3 vectors are studied for three-dimensional space. The Unit 3 vector topic leads to the establishment of the equations of lines and planes and this in turn prepares students for an introduction to solving simultaneous equations in three variables. The study of calculus, which is developed in Mathematical Methods/Specialist Methods, is applied in Vectors in Unit 3 and applications of calculus and statistics in Unit 4.

Goals

Specialist Mathematics aims to develop students':

- understanding of concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- ability to solve applied problems using concepts and techniques drawn from combinatorics, geometry, trigonometry, complex numbers, vectors, matrices, calculus and statistics
- capacity to choose and use technology appropriately.
- reasoning in mathematical and statistical contexts and interpretation of mathematical and statistical information, including ascertaining the reasonableness of solutions to problems
- capacity to communicate in a concise and systematic manner using appropriate mathematical and statistical language
- ability to construct proofs.

Student Group

Links to Foundation to Year 10

For all content areas of Specialist Mathematics, the proficiency strands of the F–10 curriculum are still very much applicable and should be inherent in students' learning of the subject. The strands of Understanding, Fluency, Problem solving and Reasoning are essential and mutually reinforcing. For all content areas, practice allows students to achieve fluency in skills, such as finding the scalar product of two vectors, or finding the area of a region contained between curves. Achieving fluency in skills such as these allows students to concentrate on more complex aspects of problem solving. In Specialist Mathematics, the formal explanation of reasoning through mathematical proof takes an important role, and the ability to present the solution of any problem in a logical and clear manner is of paramount significance. The ability to transfer skills learned to solve one class of problems, such as integration, to solve another class of problems, such as those in biology, kinematics or statistics, is a vital part of mathematics learning in this subject. In order to study Specialist Mathematics, it is desirable that students complete topics from 10A. The knowledge and skills from the following content descriptions from 10A are highly recommended as preparation for Specialist Mathematics:

- Establish the sine, cosine and area rules for any triangle, and solve related problems
- Use the unit circle to define trigonometric functions, and graph them with and without the use of digital technologies
- Investigate the concept of a polynomial and apply the factor and remainder theorems to solve problems.

Organisation of Content

Specialist Mathematics provides opportunities, beyond those presented in Mathematical Methods, to develop rigorous mathematical arguments and proofs, and to use mathematical models more extensively. Specialist Mathematics contains topics in functions and calculus that build on and deepen the ideas presented in Mathematical Methods/Specialist Methods as well as demonstrate their application in many areas. Specialist Mathematics also extends understanding and knowledge of probability and statistics and introduces the topics of vectors, complex numbers and matrices. Specialist Mathematics is the only mathematics subject that cannot be taken as a stand-alone subject.

Specialist Mathematics is structured over four units. The topics in Unit 1 broaden students' mathematical experience and provide different scenarios for incorporating mathematical arguments and problem solving. The unit provides a blending of algebraic and geometric thinking. In this subject there is a progression of content, applications, level of sophistication and abstraction. For example, vectors in the plane are introduced in Unit 1 and then in Unit 3 they are studied for three-dimensional space. In Unit 3, the topic 'Vectors in three dimensions' leads to the establishment of the equations of lines and planes, and this in turn prepares students for solving simultaneous equations in three variables.

	Unit 1	**Unit 2**	**Unit 3**	**Unit 4**
Specialist Mathematics	• Combinatorics • Vectors in the plane • Geometry	• Trigonometry • Matrices • Real and complex numbers	• Complex numbers • Functions and sketching graphs • Vectors in three dimensions	• Integration and applications of integration • Rates of change and differential equations • Statistical inference

Unit Titles

Unit 1

Unit 1 contains three topics that complement the content of Mathematical Methods/Specialist Methods. The proficiency strand, 'Reasoning', of the F–10 curriculum is continued explicitly in the topic 'Geometry' through a discussion of developing mathematical arguments. This topic also provides the opportunity to summarise and extend students' studies in Euclidean Geometry, knowledge which is of great benefit in the later study of topics such as vectors and complex numbers. The topic 'Combinatorics' provides techniques that are very useful in many areas of mathematics, including probability and algebra. The topic 'Vectors in the plane' provides new perspectives on working with two-dimensional space and serves as an introduction to techniques which can be extended to three-dimensional space in Unit 3. These three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Unit 2

Unit 2 contains three topics: ‘Trigonometry’, ‘Matrices’ and ‘Real and complex numbers’. ‘Matrices’ provides new perspectives for working with two-dimensional space, ‘Real and complex numbers’ provides a continuation of the study of numbers. The topic ‘Trigonometry’ contains techniques that are used in other topics in both this unit and Units 3 and 4. All of these topics develop students’ ability to construct mathematical arguments. The technique of proof by the principle of mathematical induction is introduced in this unit.

Unit 3

Unit 3 contains three topics: ‘Complex numbers’, ‘Vectors in three dimensions’, and ‘Functions and sketching graphs’. The Cartesian form of complex numbers was introduced in Unit 2, and in Unit 3 the study of complex numbers is extended to the polar form. The study of functions and techniques of calculus begun in Mathematical Methods/Specialist Methods is extended and utilised in the sketching of graphs and the solution of problems involving integration. The study of vectors begun in Unit 1, which focused on vectors in one- and two-dimensional space, is extended in Unit 3 to three-dimensional vectors, vector equations and vector calculus, with the latter building on students’ knowledge of calculus from Mathematical Methods/Specialist Methods. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space.

Unit 4

Unit 4 contains three topics: ‘Integration and applications of integration’, ‘Rates of change and differential equations’ and ‘Statistical inference’. In this unit, the study of differentiation and integration of functions is continued, and the techniques developed from this and previous topics in calculus are applied to the area of simple differential equations, in particular in biology and kinematics. These topics serve to demonstrate the applicability of the mathematics learnt throughout this course. Also in this unit, all of the students’ previous experience in statistics is drawn together in the study of the distribution of sample means. This is a topic that demonstrates the utility and power of statistics.

Assessment

The identification of criteria within the achievement standards and assessment task types and weightings provides a common and agreed basis for the collection of evidence of student achievement.

Assessment Criteria (the dimensions of quality that teachers look for in evaluating student work) provide a common and agreed basis for judgement of performance against unit and course goals, within and across colleges. Over a course, teachers must use all these criteria to assess students' performance but are not required to use all criteria on each task. Assessment criteria are to be used holistically on a given task and in determining the unit grade.

Assessment Tasks elicit responses that demonstrate the degree to which students have achieved the goals of a unit based on the assessment criteria. The Common Curriculum Elements (CCE) is a guide to developing assessment tasks that promote a range of thinking skills (see Appendix C). It is highly desirable that assessment tasks engage students in demonstrating higher order thinking.

Rubrics are constructed for individual tasks, informing the assessment criteria relevant for a particular task and can be used to assess a continuum that indicates levels of student performance against each criterion.

Assessment Criteria

Students will be assessed on the degree to which they demonstrate an understanding of:

- concepts and techniques
- reasoning and communications.

Assessment Task Types

Suggested tasks:	
• project/assignment • modelling projects • portfolio • journal • validation activity	• presentation such as a pitch, poster, vodcast, interview • practical activity such as a demonstration • test/examination • online adaptive tasks/quiz
Weightings in T 1.0 Units: No task to be weighted more than 50% for a standard 1.0 unit.	

Additional Assessment Information

Requirements

- For a standard unit (1.0), students must complete a minimum of three assessment tasks and a maximum of five.
- For a half standard unit (0.5), students must complete a minimum of two and a maximum of three assessment tasks.
- Students should experience a variety of task types (test and non-test) and different modes of communication to demonstrate the Achievement Standards.
- Students are required to undertake at least one problem solving investigation task each semester. This task may be completed individually or collaboratively. They are required to plan, enquire into and draw conclusions about key unit concepts. Students may respond in forms such as modelling projects, problem solving and practical activities.
- Assessment tasks for a standard (1.0) or half-standard (0.5) unit must be informed by the Achievement Standards.

Advice

- It is recommended that the total component of unsupervised tasks be no greater than 30%.
- For tasks completed in unsupervised conditions, schools need to have mechanisms to uphold academic integrity, for example, student declaration, plagiarism software, oral defence, interview, other validation tasks.

Achievement Standards

Years 11 and 12 achievement standards are written for A-T courses. A single achievement standard is written for M courses.

A Year 12 student in any unit is assessed using the Year 12 achievement standards. A Year 11 student in any unit is assessed using the Year 11 achievement standards. Year 12 achievement standards reflect higher expectations of student achievement compared to the Year 11 achievement standards. Years 11 and 12 achievement standards are differentiated by cognitive demand, the number of dimensions and the depth of inquiry.

An achievement standard cannot be used as a rubric for an individual assessment task. Assessment is the responsibility of the college. Student tasks may be assessed using rubrics or marking schemes devised by the college. A teacher may use the achievement standards to inform development of rubrics. The verbs used in achievement standards may be reflected in the rubric. In the context of combined Years 11 and 12 classes, it is best practice to have a distinct rubric for Years 11 and 12. These rubrics should be available for students prior to completion of an assessment task so that success criteria are clear.

Student achievement in A, T and M units is reported based on system standards as an A-E grade. Grade descriptors and standard work samples where available, provide a guide for teacher judgement of students' achievement over the unit.

Grades are awarded on the proviso that the assessment requirements have been met. Teachers will consider, when allocating grades, the degree to which students demonstrate their ability to complete and submit tasks within a specified time frame.

Achievement Standards for Mathematics T Course – Year 11

	*A student who achieves an **A** grade typically*	*A student who achieves a **B** grade typically*	*A student who achieves a **C** grade typically*	*A student who achieves a **D** grade typically*	*A student who achieves an **E** grade typically*
Concepts and Techniques	• critically applies mathematical concepts in a variety of complex contexts to routine and non-routine problems	• applies mathematical concepts in a variety of contexts to routine and non-routine problems	• applies mathematical concepts in some contexts to routine and non-routine problems	• applies simple mathematical concepts in limited contexts to routine problems	• applies simple mathematical concepts in structured contexts
	• selects and applies advanced mathematical techniques to solve complex problems in a variety of contexts	• selects and applies mathematical techniques to solve routine and non-routine problems in a variety of contexts	• applies mathematical techniques to solve routine and non-routine problems in some contexts	• applies simple mathematical techniques to solve routine problems in limited contexts	• uses simple mathematical techniques to solve routine problems in structured contexts
	• constructs, selects and applies complex mathematical models to routine and non-routine problems in a variety of contexts	• selects and applies mathematical models to routine and non-routine problems to a variety of contexts	• applies mathematical models to routine and non-routine problems in some contexts	• applies simple mathematical models to routine problems in limited contexts	• demonstrates limited familiarity with mathematical models in structured contexts
	• uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts	• uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts	• uses digital technologies appropriately to solve routine and non-routine problems in some contexts	• uses digital technologies appropriately to solve routine problems in limited contexts	• uses digital technologies to solve routine problems in structured contexts
Reasoning and Communications	• represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts	• represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems a variety of contexts	• represents mathematical concepts in numerical, graphical and symbolic form to some routine and some non-routine problems in some contexts	• represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in limited contexts	• represents simple mathematical concepts in numerical, graphical or symbolic form in structured contexts
	• communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and well-reasoned, using appropriate and accurate language	• communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language	• communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language	• communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language	• communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language
	• evaluates the reasonableness of solutions to routine and non-routine problems in a variety of contexts	• analyses the reasonableness of solutions to routine and non-routine problems	• explains the reasonableness of solutions to some routine and non-routine problems	• describes the appropriateness of solutions to routine problems	• identifies solutions to routine problems in structured contexts
	• reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively
	• evaluates the potential of Mathematics to generate knowledge in the public good	• analyses the potential of Mathematics to generate knowledge in the public good	• explains the potential of Mathematics to generate knowledge in the public good	• describes the potential of Mathematics to generate knowledge in the public good	• identifies some ways in which Mathematics is used to generate knowledge in the public good

Achievement Standards for Mathematics T Course – Year 12

	*A student who achieves an **A** grade typically*	*A student who achieves a **B** grade typically*	*A student who achieves a **C** grade typically*	*A student who achieves a **D** grade typically*	*A student who achieves an **E** grade typically*
Concepts and Techniques	• critically and creatively applies mathematical concepts in a variety of complex contexts to routine and non-routine problems	• critically applies mathematical concepts in a variety of contexts to routine and non-routine problems	• applies mathematical concepts in some contexts to routine and non-routine problems	• applies simple mathematical concepts in limited contexts to routine problems	• applies simple mathematical concepts in structured contexts
	• synthesises information to select and apply mathematical techniques to solve complex problems in a variety of contexts	• analyses information to select and apply mathematical techniques to solve routine and non-routine problems in a variety of contexts	• selects and applies mathematical techniques to solve routine and some non-routine problems in some contexts	• applies simple mathematical techniques to solve routine problems in limited contexts	• uses simple mathematical techniques to solve routine problems in structured contexts
	• constructs, selects and applies mathematical models to a variety of contexts in routine and non-routine problems	• selects and applies mathematical models to routine and non-routine problems in a variety of contexts	• applies mathematical models to routine and non-routine problems in some contexts	• applies simple mathematical models to routine problems in limited contexts	• demonstrates limited familiarity with mathematical models to solve routine problems in structured contexts
	• uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts	• uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts	• uses digital technologies appropriately to solve routine and non-routine problems in a variety of contexts	• uses digital technologies appropriately to solve routine problems in limited contexts	• uses digital technologies to solve routine problems in structured contexts
Reasoning and Communications	• represents complex mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts	• represents mathematical concepts in numerical, graphical and symbolic form in routine and non-routine problems in a variety of contexts	• represents mathematical concepts in numerical, graphical and symbolic form in some routine and non-routine problems in some contexts	• represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in structured contexts	• represents simple mathematical concepts in numerical, graphical or symbolic form in in simple problems in structured contexts
	• communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and reasoned, using appropriate and accurate language	• communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language	• communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language	• communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language	• communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language
	• evaluates the solutions to routine and non-routine problems in a variety of contexts	• analyses the solutions to routine and non-routine problems in some contexts	• explains solutions to some routine and non-routine problems in some contexts	• describes solutions to routine problems in limited contexts	• identifies solutions to routine problems in structured contexts
	• evaluates methods and models for their strengths and limitations when developing solutions to routine and non-routine problems	• analyses strengths and limitations of models used when developing solutions to routine and non-routine problems	• explains strengths and limitations of models used when developing solutions to some routine and non-routine problems	• describes strengths or limitations of simple models when solving routine problems	• identifies strengths or limitations of simple models in relation to routine problems
	• reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively	• reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively
	• evaluates the potential of Mathematics to generate knowledge in the public good	• analyses the potential of Mathematics to generate knowledge in the public good	• explains the potential of Mathematics to generate knowledge in the public good	• describes the potential of Mathematics to generate knowledge in the public good	• identifies some ways in which Mathematics is used to generate knowledge in the public good

Unit 1: Specialist Mathematics Value: 1.0

Unit 1a: Specialist Mathematics Value: 0.5

Unit 1b: Specialist Mathematics Value: 0.5

Corequisites

Students studying this course must also be studying the Specialist Methods or Mathematical Methods (integrating Australian Curriculum) course.

Unit Description

Unit 1 of Specialist Mathematics contains three topics – 'Combinatorics', 'Vectors in the plane' and 'Geometry' – that complement the content of Mathematical Methods. The proficiency strand, Reasoning, of the F–10 curriculum is continued explicitly in 'Geometry' through a discussion of developing mathematical arguments. While these ideas are illustrated through deductive Euclidean geometry in this topic, they recur throughout all of the topics in Specialist Mathematics. 'Geometry' also provides the opportunity to summarise and extend students' studies in Euclidean Geometry. An understanding of this topic is of great benefit in the study of later topics in the course, including vectors and complex numbers.

'Vectors in the plane' provides new perspectives for working with two-dimensional space and serves as an introduction to techniques that will be extended to three-dimensional space in Unit 3.

'Combinatorics' provides techniques that are useful in many areas of mathematics including probability and algebra. All these topics develop students' ability to construct mathematical arguments.

These three topics considerably broaden students' mathematical experience and therefore begin an awakening to the breadth and utility of the subject. They also enable students to increase their mathematical flexibility and versatility.

Access to technology to support the computational aspects of these topics is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in combinatorics, geometry and vectors
- apply reasoning skills and solve problems in combinatorics, geometry and vectors
- communicate their arguments and strategies when solving problems
- construct proofs in a variety of contexts including algebraic and geometric
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Combinatorics

Permutations (ordered arrangements):

- solve problems involving permutations
- use the multiplication principle
- use factorial notation
- solve problems involving permutations and restrictions with or without repeated objects.

The inclusion-exclusion principle for the union of two sets and three sets:

- determine and use the formulas for finding the number of elements in the union of two and the union of three sets.

The pigeon-hole principle:

- solve problems and prove results using the pigeon-hole principle.

Combinations (unordered selections):

- solve problems involving combinations
- use the notation $\binom{n}{r}$ or ${}^{n}C_{r}$
- derive and use simple identities associated with Pascal's triangle.

Topic 2: Vectors in the plane

Representing vectors in the plane by directed line segments:

- examine examples of vectors including displacement and velocity
- define and use the magnitude and direction of a vector
- represent a scalar multiple of a vector
- use the triangle rule to find the sum and difference of two vectors.

Algebra of vectors in the plane:

- use ordered pair notation and column vector notation to represent a vector
- define and use unit vectors and the perpendicular unit vectors i and j
- express a vector in component form using the unit vectors i and j
- examine and use addition and subtraction of vectors in component form
- define and use multiplication by a scalar of a vector in component form
- define and use scalar (dot) product
- apply the scalar product to vectors expressed in component form
- examine properties of parallel and perpendicular vectors and determine if two vectors are parallel or perpendicular
- define and use projections of vectors
- solve problems involving displacement, force and velocity involving the above concepts.

Topic 3: Geometry

The nature of proof:

- use implication, converse, equivalence, negation, contrapositive
- use proof by contradiction
- use the symbols for implication ($\Rightarrow$), equivalence ($\Leftrightarrow$), and equality (=)
- use the quantifiers ‘for all’ and ‘there exists’
- use examples and counter-examples.

Circle properties and their proofs including the following theorems:

- an angle in a semicircle is a right angle
- the angle at the centre subtended by an arc of a circle is twice the angle at the circumference subtended by the same arc
- angles at the circumference of a circle subtended by the same arc are equal
- the opposite angles of a cyclic quadrilateral are supplementary
- chords of equal length subtend equal angles at the centre and conversely chords subtending equal angles at the centre of a circle have the same length
- the alternate segment theorem
- when two chords of a circle intersect, the product of the lengths of the intervals on one chord equals the product of the lengths of the intervals on the other chord
- when a secant (meeting the circle at A and B) and a tangent (meeting the circle at T) are drawn to a circle from an external point M, the square of the length of the tangent equals the product of the lengths to the circle on the secant. ($AM \times BM = TM^2$)
- suitable converses of some of the above results
- solve problems finding unknown angles and lengths and prove further results using the results listed above

Geometric proofs using vectors in the plane including:

- the diagonals of a parallelogram meet at right angles if and only if it is a rhombus
- midpoints of the sides of a quadrilateral join to form a parallelogram
- the sum of the squares of the lengths of the diagonals of a parallelogram is equal to the sum of the squares of the lengths of the sides.

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 11-13.

Unit 2: Specialist Mathematics — Value: 1.0

Unit 2a: Specialist Mathematics — **Value: 0.5**

Unit 2b: Specialist Mathematics — **Value: 0.5**

Corequisites

Students studying this course must also be studying the Specialist Methods or Mathematical Methods (integrating Australian Curriculum) course.

Unit Description

Unit 2 of Specialist Mathematics contains three topics – 'Trigonometry', 'Real and complex numbers' and 'Matrices'…'Trigonometry' contains techniques that are used in other topics in both this unit and Unit 3. 'Real and complex numbers' provides a continuation of students' study of numbers, and the study of complex numbers is continued in Unit 3. This topic also contains a section on proof by mathematical induction. The study of matrices is undertaken, including applications to linear transformations of the plane. Access to technology to support the computational aspects of these topics is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in trigonometry, real and complex numbers, and matrices
- apply reasoning skills and solve problems in trigonometry, real and complex numbers, and matrices
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Trigonometry

The basic trigonometric functions:

- find all solutions of $f\big(a(x-b)\big)=c$ where f is one of sin, cos or tan
- graph functions with rules of the form $y=f(a(x-b))$ where f is one of sin, cos, or tan.

Compound angles:

- prove and apply the angle sum, difference and double angle identities.

The reciprocal trigonometric functions, secant, cosecant and cotangent:

- define the reciprocal trigonometric functions, sketch their graphs, and graph simple transformations of them.

Trigonometric identities:

- prove and apply the Pythagorean identities
- prove and apply the identities for products of sines and cosines expressed as sums and differences
- convert sums $a \cos x + b \sin x$ to $R \cos(x \pm \alpha)$ or $R \sin(x \pm \alpha)$ and apply these to sketch graphs, solve equations of the form $a \cos x + b \sin x = c$ and solve problems
- prove and apply other trigonometric identities such as $\cos 3x = 4 \cos^3 x - 3 \cos x$.

Applications of trigonometric functions to model periodic phenomena:

- model periodic motion using sine and cosine functions and understand the relevance of the period and amplitude of these functions in the model.

Topic 2: Matrices

Matrix arithmetic:

- understand the matrix definition and notation
- define and use addition and subtraction of matrices, scalar multiplication, matrix multiplication, multiplicative identity and inverse
- calculate the determinant and inverse of 2 × 2 matrices and solve matrix equations of the form

  AX = B, where A is a 2 × 2 matrix and X and B are column vectors.

Transformations in the plane:

- translations and their representation as column vectors
- define and use basic linear transformations: dilations of the form $(x, y) \longrightarrow (\lambda_1 x, \lambda_2 y)$, rotations about the origin and reflection in a line which passes through the origin, and the representations of these transformations by 2×2 matrices
- apply these transformations to points in the plane and geometric objects
- define and use composition of linear transformations and the corresponding matrix products
- define and use inverses of linear transformations and the relationship with the matrix inverse
- examine the relationship between the determinant and the effect of a linear transformation on area
- establish geometric results by matrix multiplications; for example, show that the combined effect of two reflections in lines through the origin is a rotation.

Topic 3: Real and complex numbers

Proofs involving numbers:

- prove simple results involving numbers.

Rational and irrational numbers:

- express rational numbers as terminating or eventually recurring decimals and vice versa
- prove irrationality by contradiction for numbers such as $\sqrt{2}$ and $\log_2 5$.

An introduction to proof by mathematical induction:

- understand the nature of inductive proof including the 'initial statement' and inductive step
- prove results for sums, such as $1 + 4 + 9 \ldots + n^2 = \frac{n(n+1)(2n+1)}{6}$ for any positive integer n
- prove divisibility results, such as $3^{2n+4} - 2^{2n}$ is divisible by 5 for any positive integer n.

Complex numbers

- define the imaginary number i as a root of the equation $x^2 = -1$
- use complex numbers in the form a + bi where a and b are the real and imaginary parts

- determine and use complex conjugates
- perform complex-number arithmetic: addition, subtraction, multiplication and division.

The complex plane:

- consider complex numbers as points in a plane with real and imaginary parts as Cartesian coordinates
- examine addition of complex numbers as vector addition in the complex plane
- understand and use location of complex conjugates in the complex plane.

Roots of equations:

- use the general solution of real quadratic equations
- determine complex conjugate solutions of real quadratic equations
- determine linear factors of real quadratic polynomials.

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 11-13.

Unit 3: Specialist Mathematics Value: 1.0

Unit 3a: Specialist Mathematics Value: 0.5

Unit 3b: Specialist Mathematics Value: 0.5

Corequisites

Students studying this course must also be studying the Specialist Methods or Mathematical Methods (integrating Australian Curriculum) course.

Unit Description

Unit 3 of Specialist Mathematics contains three topics: 'Vectors in three dimensions', 'Complex numbers' and 'Functions and sketching graphs'. The study of vectors was introduced in Unit 1 with a focus on vectors in two-dimensional space. In this unit, three-dimensional vectors are studied and vector equations and vector calculus are introduced, with the latter extending students' knowledge of calculus from Mathematical Methods. Cartesian and vector equations, together with equations of planes, enables students to solve geometric problems and to solve problems involving motion in three-dimensional space. The Cartesian form of complex numbers was introduced in Unit 2, and the study of complex numbers is now extended to the polar form. The study of functions and techniques of graph sketching, begun in Mathematical Methods, is extended and applied in sketching graphs and solving problems involving integration. Access to technology to support the computational aspects of these topics is assumed.

Specific Unit Goals

By the end of this unit, students will:

- understand the concepts and techniques in vectors, complex numbers, functions and graph sketching
- apply reasoning skills and solve problems in vectors, complex numbers, functions and graph sketching
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical information and ascertain the reasonableness of their solutions to problems

Content Descriptions

Further elaboration on the content of this unit is available at: Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Complex numbers

Cartesian forms:

- review real and imaginary parts $Re(z)$ and $Im(z)$ of a complex number z
- review Cartesian form
- review complex arithmetic using Cartesian forms.

Complex arithmetic using polar form:

- use the modulus $|z|$of a complex number z and the argument $\text{Arg}\ (z)$ of a non-zero complex number z and prove basic identities involving modulus and argument
- convert between Cartesian and polar form
- define and use multiplication, division, and powers of complex numbers in polar form and the geometric interpretation of these
- prove and use De Moivre's theorem for integral powers.

The complex plane (the Argand plane):

- examine and use addition of complex numbers as vector addition in the complex plane
- examine and use multiplication as a linear transformation in the complex plane
- identify subsets of the complex plane determined by relations such as$|z - 3i| \leq 4, \frac{\pi}{4} \leq \text{Arg}(z) \leq \frac{3\pi}{4}$, $\text{Re}(z) > Im(z)$, and $|z - 1| = 2|z - i|$.

Roots of complex numbers

- determine and examine the n^{th} roots of unity and their location on the unit circle
- determine and examine the n^{th}roots of complex numbers and their location in the complex plane.

Factorisation of polynomials:

- prove and apply the factor theorem and the remainder theorem for polynomials
- consider conjugate roots for polynomials with real coefficients (
- solve simple polynomial equations.

Topic 2: Functions and sketching graphs

Functions:

- determine when the composition of two functions is defined
- find the composition of two functions
- determine if a function is one-to-one (
- consider inverses of a one-to-one functions
- examine the reflection property of the graph of a function and the graph of its inverse.

Sketching graphs:

- use and apply the notation $|x|$ for the absolute value for the real number x and the graph of $y = |x|$
- examine the relationship between the graph of $y = f(x)$ and the graphs of $y = \frac{1}{f(x)}$, $y = |f(x)|$ and $y = f(|x|)$
- sketch the graphs of simple rational functions where the numerator and denominator are polynomials of low degree.

Topic 3: Vectors in three dimensions

The algebra of vectors in three dimensions:

- review the concepts of vectors from Unit 1 and extend to three dimensions including introducing the unit vectors i, j and k
- prove geometric results in the plane and construct simple proofs in three-dimensions.

Vector and Cartesian equations:

- introduce Cartesian coordinates for three-dimensional space, including plotting points and the equations of spheres
- use vector equations of curves in two or three dimensions involving a parameter, and determine a ‘corresponding’ Cartesian equation in the two-dimensional case
- determine a vector equation of a straight line and straight-line segment, given the position of two points, or equivalent information, in both two and three dimensions
- examine the position of two particles each described as a vector function of time, and determine if their paths cross or if the particles meet
- use the cross product to determine a vector normal to a given plane
- determine vector and Cartesian equations of a plane and of regions in a plane.

Systems of linear equations:

- recognise the general form of a system of linear equations in several variables, and use elementary techniques of elimination to solve a system of linear equations
- examine the three cases for solutions of systems of equations – a unique solution, no solution, and infinitely many solutions – and the geometric interpretation of a solution of a system of equations with three variables.

Vector calculus:

- consider position of vectors as a function of time
- derive the Cartesian equation of a path given as a vector equation in two dimensions including ellipses and hyperbolas
- differentiate and integrate a vector function with respect to time
- determine equations of motion of a particle travelling in a straight line with both constant and variable acceleration
- apply vector calculus to motion in a plane including projectile and circular motion.

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 11-13.

Unit 4: Specialist Mathematics Value: 1.0

Unit 4a: Specialist Mathematics Value: 0.5

Unit 4b: Specialist Mathematics Value: 0.5

Corequisites

Students studying this course must also be studying the Specialist Methods or Mathematical Methods (integrating Australian Curriculum) course.

Unit Description

Unit 4 of Specialist Mathematics contains three topics: 'Integration and applications of integration', 'Rates of change and differential equations' and 'Statistical inference'. In Unit 4, the study of differentiation and integration of functions continues, and the calculus techniques developed in this and previous topics are applied to simple differential equations, in particular in biology and kinematics. These topics demonstrate the real-world applications of the mathematics learned throughout Specialist Mathematics. In this unit all of the students' previous experience working with probability and statistics is drawn together in the study of statistical inference for the distribution of sample means and confidence intervals for sample means. Access to technology to support the computational aspects of these topics is assumed.

Specific Unit Goals

By the end of this unit, students:

- understand the concepts and techniques in applications of calculus and statistical inference
- apply reasoning skills and solve problems in applications of calculus and statistical inference
- communicate their arguments and strategies when solving problems
- construct proofs of results
- interpret mathematical and statistical information and ascertain the reasonableness of their solutions to problems.

Content Descriptions

Further elaboration of the content of this unit is available on the ACARA Australian Curriculum website.

Topic 1: Integration and applications of integration

Integration techniques:

- integrate using the trigonometric identities $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$ and $1 + \tan^2 x = \sec^2 x$
- use substitution $u = g(x)$ to integrate expressions of the form $f\big(g(x)\big)g'(x)$
- establish and use the formula $\int \frac{1}{x} dx = \ln\ |x\,| + c$, for $x \neq$
- find and use the inverse trigonometric functions: arcsine, arccosine and arctangent
- find and use the derivative of the inverse trigonometric functions: arcsine, arccosine and arctangent
- integrate expressions of the form $\frac{\pm 1}{\sqrt{a^2 - x^2}}$ and $\frac{a}{a^2 + x^2}$
- use partial fractions where necessary for integration in simple cases
- integrate by parts.

Applications of integral calculus:

- calculate areas between curves determined by functions

- determine volumes of solids of revolution about either axis
- use numerical integration using technology
- use and apply the probability density function, $f(t) = \lambda e^{-\lambda t}$ for $t \geq 0$, of the exponential random variable with parameter $\lambda > 0$, and use the exponential random variables and associated probabilities and quantiles to model data and solve practical problems.

Topic 2: Rates of change and differential equations

- use implicit differentiation to determine the gradient of curves whose equations are given in implicit form
- Related rates as instances of the chain rule: $\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$
- solve simple first-order differential equations of the form $\frac{dy}{dx} = f(x)$, differential equations of the form $\frac{dy}{dx} = g(y)$ and, in general, differential equations of the form $\frac{dy}{dx} = f(x)g(y)$ using separation of variables
- examine slope (direction or gradient) fields of a first order differential equation
- formulate differential equations including the logistic equation that will arise in, for example, chemistry, biology and economics, in situations where rates are involved.

Modelling motion:

- examine momentum, force, resultant force, action and reaction
- consider constant and non-constant force
- understand motion of a body under concurrent forces
- consider and solve problems involving motion in a straight line with both constant and non-constant acceleration, including simple harmonic motion and the use of expressions $\frac{dv}{dt}$, $v\frac{dv}{dx}$ and $\frac{d(\frac{1}{2}v^2)}{dx}$ for acceleration.

Topic 3: Statistical inference

Sample means:

- examine the concept of the sample mean $\bar{X}$ as a random variable whose value varies between samples where X is a random variable with mean μ and the standard deviation σ
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate properties of the distribution of $\bar{X}$ across samples of a fixed size n, including its mean μ, its standard deviation $\sigma/\sqrt{n}$ (where μ and σ are the mean and standard deviation of X), and its approximate normality if n is large
- simulate repeated random sampling, from a variety of distributions and a range of sample sizes, to illustrate the approximate standard normality of $\frac{\bar{X}-\mu}{s/\sqrt{n}}$ for large samples ($n \geq 30$), where s is the sample standard deviation.

Confidence intervals for means:

- understand the concept of an interval estimate for a parameter associated with a random variable
- examine the approximate confidence interval $\left(\bar{X} - \frac{zs}{\sqrt{n}},\ \bar{X} + \frac{zs}{\sqrt{n}}\right)$, as an interval estimate for μ ,the population mean, where z is the appropriate quantile for the standard normal distribution
- use simulation to illustrate variations in confidence intervals between samples and to show that most but not all confidence intervals contain μ
- use $\bar{x}$ and s to estimate μ and σ, to obtain approximate intervals covering desired proportions of values of a normal random variable and compare with an approximate confidence interval for μ

- collect data and construct an approximate confidence interval to estimate a mean and to report on survey procedures and data quality.

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 11-13.

Appendix A – Implementation Guidelines

Available course patterns

A standard 1.0 value unit is delivered over at least 55 hours. To be awarded a course, students must complete at least the minimum units over the whole minor, major, major/minor or double major course.

Course	Number of standard units to meet course requirements
Major-Minor	Minimum of 5.5 units
Double Major	Minimum of 7 units

It is recommended that Units 1-4 are studied sequentially.

A minor or major are not available in this course.

The following rules apply where students have studied units from the Specialist Mathematics course:

- a Specialist Mathematics major-minor consists of a major (3.5 - 4 units) in Specialist Methods and a total of 2.0-3.0 units of Specialist Mathematics, depending on the college.
- a Specialist Mathematics double major consists of a major (3.5 or 4 units) in Specialist Methods and a minimum of 3.0 units of Specialist Mathematics, with the total number of units required depending on the college.
- students who complete a major in Specialist Methods and fewer than 2.0 units of Specialist Mathematics will include these units in a course in Specialist Methods.

Prerequisites for the course or units within the course

Students may study Specialist Methods T concurrently with Specialist Mathematics T integrating Australian Curriculum.

Arrangements for students continuing study in this course

Students who studied the previous course may undertake any units in this course provided there is no duplication of content.

Duplication of Content Rules

Students cannot be given credit towards the requirements for a Senior Secondary Certificate for a unit that significantly duplicates content in a unit studied in another course. The responsibility for preventing undesirable overlap of content studied by a student rests with the principal and the teacher delivering the course. Students will only be given credit for covering the content once.

Guidelines for Delivery

Program of Learning

A program of learning is what a school provides to implement the course for a subject. This meets the requirements for context, scope and sequence set out in the Board endorsed course. Students follow programs of learning in a college as part of their senior secondary studies. The detail, design and layout of a program of learning are a college decision.

The program of learning must be documented to show the planned learning activities and experiences that meet the needs of particular groups of students, taking into account their interests, prior knowledge, abilities and backgrounds. The program of learning is a record of the learning experiences that enable students to achieve the knowledge, understanding and skills of the content descriptions. There is no requirement to submit a program of learning to the OBSSS for approval. The Principal will need to sign off at the end of Year 12 that courses have been delivered as accredited.

Content Descriptions

Are all content descriptions of equal importance? No. It depends on the focus of study. Teachers can customise their program of learning to meet their own students' needs, adding additional content descriptions if desired or emphasising some over others. A teacher must balance student needs with their responsibility to teach all content descriptions. It is mandatory that teachers address all content descriptions and that students engage with all content descriptions.

Half standard 0.5 units

Half standard units appear on the course adoption form but are not explicitly documented in courses. It is at the discretion of the college principal to split a standard 1.0 unit into two half standard 0.5 units. Colleges are required to adopt the half standard 0.5 units. However, colleges are not required to submit explicit documentation outlining their half standard 0.5 units to the BSSS. Colleges must assess students using the half standard 0.5 assessment task weightings outlined in the framework. It is the responsibility of the college principal to ensure that all content is delivered in units approved by the Board.

System Moderation

System moderation begins in schools whereby teachers cooperate to develop assessment, and grade and score student assessment according to the relevant curriculum.

Moderation Day is an essential component of the ACT senior secondary system which empowers school autonomy in curriculum and assessment. Moderation Day is a collaborative and professional event whereby schools undertake system quality assurance activities on behalf of their current and future students. Moderation Day fosters and enriches the development of quality assessment and validates student achievement. Continued best practice in teaching and learning is ensured through the formation of valid, constructive, and detailed feedback.

System Moderation:

- provides comparability of school-based assessment
- forms the basis for valid and reliable assessment in senior secondary schools
- involves the ACT Board of Senior Secondary Studies (BSSS) and schools in cooperation and partnership
- maintains the integrity of the ACT Senior Secondary Certificate.

The Moderation Model

Moderation within the ACT senior secondary system encompasses structured, consensus-based peer review of Unit Grades and quality of assessment for all BSSS courses twice per year. In addition to System Moderation, there is statistical moderation of course scores.

Moderation by Structured, Consensus-based Peer Moderation

Consensus-based peer moderation involves the review of student assessment against system wide criteria and standards and the validation of Unit Grades. This is done by matching student performance with the Framework Achievement Standards. In addition, feedback will be provided on the quality of the task.

Preparation for Structured, Consensus-based Peer Review

Schools retain originals or copies of student assessment evidence completed in the delivery of the unit and all unit documentation. Student assessment evidence must be sufficient to allow reviewing teachers to make an accurate judgment of grade standard. Schools will use ACS to present this information for System Moderation. Criteria for each Moderation Day will be communicated to schools in the proceeding calendar year.

Feedback from System Moderation

Feedback is provided to schools to affirm good practice and inform continuous improvement. This feedback is based on the BSSS Quality Assessment Guidelines and relevant course documents. It is expected that schools engage with feedback and address any longitudinal trends as outlined in the *BSSS Policy and Procedures Manual*.

Appendix B – Course Developers

Name	College
Jacob Woolley	Canberra College
Gary Pocock	Canberra Institute of Technology
Marion McIntosh	Melba Copland Secondary School
Wayne Semmens	Melba Copland Secondary School
Jennifer Missen	Merici College
Nicole Burg	Narrabundah College
Rebecca Guinane	Narrabundah College
Andrew Trost	Narrabundah College

Appendix C – Common Curriculum Elements

Common curriculum elements assist in the development of high-quality assessment tasks by encouraging breadth and depth and discrimination in levels of achievement.

Organisers	Elements	Examples
create, compose and apply	apply	ideas and procedures in unfamiliar situations, content and processes in non-routine settings
	compose	oral, written and multimodal texts, music, visual images, responses to complex topics, new outcomes
	represent	images, symbols or signs
	create	creative thinking to identify areas for change, growth and innovation, recognise opportunities, experiment to achieve innovative solutions, construct objects, imagine alternatives
	manipulate	images, text, data, points of view
analyse, synthesise and evaluate	justify	arguments, points of view, phenomena, choices
	hypothesise	statement/theory that can be tested by data
	extrapolate	trends, cause/effect, impact of a decision
	predict	data, trends, inferences
	evaluate	text, images, points of view, solutions, phenomenon, graphics
	test	validity of assumptions, ideas, procedures, strategies
	argue	trends, cause/effect, strengths and weaknesses
	reflect	on strengths and weaknesses
	synthesise	data and knowledge, points of view from several sources
	analyse	text, images, graphs, data, points of view
	examine	data, visual images, arguments, points of view
	investigate	issues, problems
organise, sequence and explain	sequence	text, data, relationships, arguments, patterns
	visualise	trends, futures, patterns, cause and effect
	compare/contrast	data, visual images, arguments, points of view
	discuss	issues, data, relationships, choices/options
	interpret	symbols, text, images, graphs
	explain	explicit/implicit assumptions, bias, themes/arguments, cause/effect, strengths/weaknesses
	translate	data, visual images, arguments, points of view
	assess	probabilities, choices/options
	select	main points, words, ideas in text
identify, summarise and plan	reproduce	information, data, words, images, graphics
	respond	data, visual images, arguments, points of view
	relate	events, processes, situations
	demonstrate	probabilities, choices/options
	describe	data, visual images, arguments, points of view
	plan	strategies, ideas in text, arguments
	classify	information, data, words, images
	identify	spatial relationships, patterns, interrelationships
	summarise	main points, words, ideas in text, review, draft and edit

Appendix D – Glossary of Verbs

Verbs	Definition
Analyse	Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences
Apply	Use, utilise or employ in a particular situation
Argue	Give reasons for or against something
Assess	Make a Judgement about the value of
Classify	Arrange into named categories in order to sort, group or identify
Compare	Estimate, measure or note how things are similar or dissimilar
Compose	The activity that occurs when students produce written, spoken, or visual texts
Contrast	Compare in such a way as to emphasise differences
Create	Bring into existence, to originate
Critically analyse	Analysis that engages with criticism and existing debate on the issue
Demonstrate	Give a practical exhibition an explanation
Describe	Give an account of characteristics or features
Discuss	Talk or write about a topic, taking into account different issues or ideas
Evaluate	Examine and judge the merit or significance of something
Examine	Determine the nature or condition of
Explain	Provide additional information that demonstrates understanding of reasoning and /or application
Extrapolate	Infer from what is known
Hypothesise	Put forward a supposition or conjecture to account for certain facts and used as a basis for further investigation by which it may be proved or disproved
Identify	Recognise and name
Interpret	Draw meaning from
Investigate	Planning, inquiry into and drawing conclusions about
Justify	Show how argument or conclusion is right or reasonable
Manipulate	Adapt or change
Plan	Strategize, develop a series of steps, processes
Predict	Suggest what might happen in the future or as a consequence of something
Reflect	The thought process by which students develop an understanding and appreciation of their own learning. This process draws on both cognitive and affective experience
Relate	Tell or report about happenings, events or circumstances
Represent	Use words, images, symbols or signs to convey meaning
Reproduce	Copy or make close imitation
Respond	React to a person or text
Select	Choose in preference to another or others
Sequence	Arrange in order
Summarise	Give a brief statement of the main points
Synthesise	Combine elements (information/ideas/components) into a coherent whole
Test	Examine qualities or abilities
Translate	Express in another language or form, or in simpler terms
Visualise	The ability to decode, interpret, create, question, challenge and evaluate texts that communicate with visual images as well as, or rather than, words

Appendix E – Glossary for ACT Senior Secondary Curriculum

Courses will detail what teachers are expected to teach and students are expected to learn for year 11 and 12. They will describe the knowledge, understanding and skills that students will be expected to develop for each learning area across the years of schooling.

Learning areas are broad areas of the curriculum, including English, mathematics, science, the arts, languages, health and physical education.

A **subject** is a discrete area of study that is part of a learning area. There may be one or more subjects in a single learning area.

Frameworks are system documents for Years 11 and 12 which provide the basis for the development and accreditation of any course within a designated learning area. In addition, frameworks provide a common basis for assessment, moderation and reporting of student outcomes in courses based on the framework.

The **course** sets out the requirements for the implementation of a subject. Key elements of a course include the rationale, goals, content descriptions, assessment, and achievement standards as designated by the framework.

BSSS courses will be organised into units. A unit is a distinct focus of study within a course. A standard 1.0 unit is delivered for a minimum of 55 hours generally over one semester.

Core units are foundational units that provide students with the breadth of the subject.

Additional units are avenues of learning that cannot be provided for within the four core 1.0 standard units by an adjustment to the program of learning.

An **Independent Study unit** is a pedagogical approach that empowers students to make decisions about their own learning. Independent Study units can be proposed by a student and negotiated with their teacher but must meet the specific unit goals and content descriptions as they appear in the course.

An **elective** is a lens for demonstrating the content descriptions within a standard 1.0 or half standard 0.5 unit.

A **lens** is a particular focus or viewpoint within a broader study.

Content descriptions refer to the subject-based knowledge, understanding and skills to be taught and learned.

A **program of learning** is what a college develops to implement the course for a subject and to ensure that the content descriptions are taught and learned.

Achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following completion of study of senior secondary course content for units in a subject.

ACT senior secondary system **curriculum** comprises all BSSS approved courses of study.

Appendix F – Specialist Mathematics Glossary

Unit 1

Combinatorics

Arranging *n* objects in an ordered list

The number of ways to arrange *n* different objects in an ordered list is given by

$n(n-1)(n-2) \times \ldots \times 3 \times 2 \times 1 = n!$

Combinations (Selections)

The number of selections of *n* objects taken *r* at a time (that is, the number of ways of selecting *r* objects out of *n*) is denoted by ${}^nC_r = \binom{n}{r}$ and is equal to

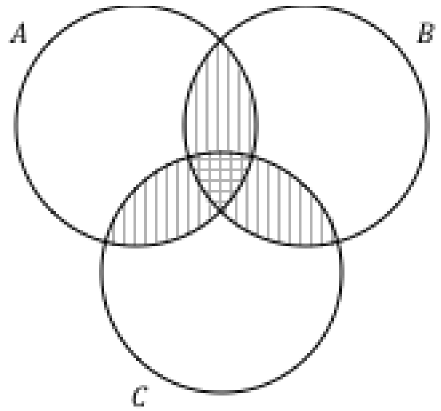
$$\frac{n!}{r!(n-r)!}$$

Inclusion – exclusion principle

- Suppose *A* and *B* are subsets of a finite set *X* then

$$|A \cup B| = |A| + |B| - |A \cap B|$$

- Suppose *A*, *B* and *C* are subsets of a finite set *X* then



$$|A \cup B \cup C| = |A| + |B| + |C| - |A \cap B| - |A \cap C| - |B \cap C| + |A \cap B \cap C|$$

This result can be generalised to 4 or more sets.

Multiplication principle

Suppose a choice is to be made in two stages. If there are *a* choices for the first stage and *b* choices for the second stage, no matter what choice has been made at the first stage, then there are *ab* choices altogether. If the choice is to be made in *n* stages and if for each *i*, there are a_i choices for the i^{th} stage then there are $a_1a_2 \ldots a_n$ choices altogether.

Pascal's triangle

Pascal's triangle is an arrangement of numbers. In general the n^{th} row consists of the binomial coefficients ^nC_r or $\binom{n}{r}$ with the r = 0, 1,…, n

				1		1				
			1		2		1			
		1		3		3		1		
	1		4		6		4		1	
1		5		10		10		5		1

In Pascal's triangle any term is the sum of the two terms 'above' it.

For example 10 = 4 + 6.

Identities include:

- The recurrence relation, $^nC_k = {}^{n-1}C_{k-1} + {}^{n-1}C_k$
- $^nC_k = \frac{n}{k}\,{}^{n-1}C_{k-1}$

Permutations

A permutation of n objects is an arrangement or rearrangement of n objects (order is important). The number of arrangements of n objects is n! The number of permutations of n objects taken r at a time is denoted ^nP_r and is equal to

$n(n-1)\ldots(n-r+1) = \frac{n!}{(n-r)!}$.

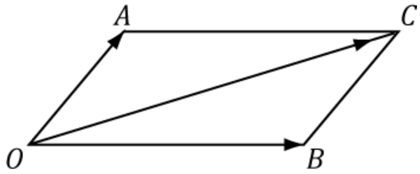
Pigeon-hole principle

If there are n pigeon holes and n + 1 pigeons to go into them, then at least one pigeon hole must get 2 or more pigeons.

Vectors in the plane

Addition of vectors (see Vector for definition and notation)

Given vectors **a** and **b** let $\overrightarrow{OA}$ and $\overrightarrow{OB}$ be directed line segments that represent **a** and **b.** They have the same initial point O. The sum of $\overrightarrow{OA}$ and $\overrightarrow{OB}$ is the directed line segment $\overrightarrow{OC}$ where C is a point such that $OACB$ is a parallelogram. This is known as the **parallelogram rule**.



If **a** = (a_1, a_2) and **b =** (b_1, b_2) then **a** + **b** = $(a_1 + b_1, a_2 + b_2)$

In component form if **a** = a_1**i +** a_2**j** and **b =** b_1**i** + b_2**j** then

a + **b** = $(a_1 + b_1)$**i +** $(a_2 + b_2)$**j**

Properties of vector addition:

- **a** + **b** = **b** + **a** (commutative law)
- (**a** + **b**) + **c** = **a** + (**b** + **c**) (associative law)
- **a** + **0** = **0** + **a** = **a**
- **a** + (–**a**) = **0**

Magnitude of a vector (see Vector for definition and notation)

The magnitude of a vector **a** is the length of any directed line segment that represents **a.** It is denoted by |**a**|.

Multiplication by a scalar

Let **a** be a non-zero vector and *k* a positive real number (scalar) then the scalar multiple of **a** by *k* is the vector *k**a*** which has magnitude |*k* ||**a**| and the same direction as **a.** If *k* is a negative real number, then *k* **a** has magnitude | *k* | |**a**| and but is directed in the opposite direction to **a**. (see **negative of a vector**)

Some properties of scalar multiplication are:

- *k* (**a** + **b**) = *k* **a** + *k* **b**
- *h*(*k* (**a**)) = (*hk*)**a**
- 1**a** = **a**

Negative of a vector (see Vector for definition and notation)

Given a vector **a**, let $\overrightarrow{AB}$ be a directed line segment representing **a.** The negative of **a**, denoted by –**a,** is the vector represented by $\overrightarrow{BA}$. The following are properties of vectors involving negatives:

- **a** + (–**a**) = (–**a**) + **a** = **0**
- –(–**a**) = **a**

Scalar product (see Vector for definition and notation)

a = (a_1, a_2) and **b** = (b_1, b_2) then the scalar product **a**.**b** is the real number

$a_1 b_1 + a_2 b_2$. The geometrical interpretation of this number is **a.b** =|**a**||**b**|cos (θ) where θ is the angle 'between' **a** and **b**

When expressed in **i**, **j**, notation, if **a** = a_1**i** + a_2**j** and **b** = b_1**i** + b_2**j** then

a.b = $a_1 b_1 + a_2 b_2$

Note |**a**| = $\sqrt{\mathbf{a}.\mathbf{a}}$

Subtraction of vectors (see Vector for definition and notation)

a – b = a + (–b)

Unit vector (see Vector for definition and notation)

A unit vector is a vector with magnitude 1. Given a vector **a,** the unit vector in the same direction as **a** is $\frac{1}{|\mathbf{a}|}$ **a.** This vector is often denoted as $\hat{\mathbf{a}}$.

Vector projection (see Vector for definition and notation)

Let **a** and **b** be two vectors and write θ for the angle between them. The projection of a vector **a** on a vector **b** is the vector

$|\mathbf{a}| \cos\theta \, \hat{\mathbf{b}}$ where $\hat{\mathbf{b}}$ is the unit vector in the direction of **b.**

The projection of a vector **a** on a vector **b** is $(\mathbf{a}.\hat{\mathbf{b}})\,\hat{\mathbf{b}}$ where $\hat{\mathbf{b}}$ is the unit vector in the direction of **b.** This projection is also given by the formula $\frac{\mathbf{a.b}}{\mathbf{b.b}}\mathbf{b}$.

Vector

In Physics the name vector is used to describe a physical quantity like velocity or force that has a magnitude and direction.

A vector is an entity **a** which has a given length (magnitude) and a given direction. If $\overrightarrow{AB}$ is a directed line segment with this length and direction, then we say that $\overrightarrow{AB}$ represents **a.**

If $\overrightarrow{AB}$ and $\overrightarrow{CD}$ represent the same vector, they are parallel and have the same length.

The **zero vector** is the vector with length zero.

In two dimensions, every vector can be represented by a directed line segment which begins at the origin.

For example, the vector $\overrightarrow{BC}$ from B(1,2) to C(5,7) can be represented by the directed line segment $\overrightarrow{OA}$, where A is the point (4,5). The **ordered pair** notation for a vector uses the co-ordinates of the end point of this directed line segment beginning at the origin to denote the vector, so

$\overrightarrow{BC}$ = (4,5) in ordered pair notation. The same vector can be represented in **column vector** notation as

$$\begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

Geometry

Glossary for Proof

Contradiction-Proof by

Assume the opposite (**negation**) of what you are trying to prove. Then proceed through a logical chain of argument till you reach a demonstrably false conclusion. Since all the reasoning is correct and a false conclusion has been reached the only thing that could be wrong is the initial assumption. Therefore the original statement is true.

For example: the result $\sqrt{2}$ is irrational can be proved in this way by first assuming $\sqrt{2}$ is rational.

The following are examples of results that are often proved by contradiction:

- If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
- If an interval (line segment) subtends equal angles at two points on the same side of the interval (line segment), then the two points and the endpoints of the interval are concyclic.

Implication and Converse

Implication: if P then Q Symbol: $P \Rightarrow Q$

Examples:

- If a quadrilateral is a rectangle, then the diagonals are of equal length and they bisect each other.
- If $x = 2$ then $x^2 = 4$.
- If an animal is a kangaroo, then it is a marsupial.
- If a quadrilateral is cyclic then the opposite angles are supplementary.

Converse of a statement The converse of the statement 'If P then Q' is 'If Q then P' Symbolically the converse of $P \Rightarrow Q$ is: $Q \Rightarrow P$ or $P \Leftarrow Q$

The converse of a true statement need not be true.

Examples:

- **Statement:** If a quadrilateral is a rectangle, then the diagonals are of equal length and they bisect each other.

  Converse statement: If the diagonals of a quadrilateral are of equal length and bisect each other than the quadrilateral is a rectangle. (In this case the converse is true.)

- **Statement:** If $x = 2$ then $x^2 = 4$.

  Converse statement: If $x^2 = 4$ then $x = 2$. (In this case the converse is false.)

- **Statement:** If an animal is a kangaroo, then it is a marsupial.

  Converse statement: If an animal is a marsupial then it is a kangaroo. (In this case the converse is false.)

- **Statement:** If a quadrilateral is cyclic then the opposite angles are supplementary.

  Converse statement: If the opposite angles of a quadrilateral are supplementary then the quadrilateral is cyclic. (In this case the converse is true.)

Contrapositive

The contrapositive of the statement 'If P then Q' is 'If not Q then not P'. The contrapositive of a true statement is also true. (not Q is the **negation** of the statement Q)

Examples:

- **Statement:** A rectangle is a quadrilateral that has diagonals of equal length and the diagonals bisect each other.

  Contrapositive: If the diagonals of a quadrilateral are not of equal length or do not bisect each other than the quadrilateral is not a rectangle.

- **Statement:** If $x = 2$ then $x^2 = 4$.

  Contrapositive: If $x^2 \neq 4$ then $x \neq 2$.

- **Statement:** A kangaroo is a marsupial.

  Contrapositive: If an animal is not a marsupial then it is not a kangaroo.

- **Statement:** The opposite angles of a cyclic quadrilateral are supplementary

  Contrapositive: If the opposite angles of quadrilateral are not supplementary then the quadrilateral is not cyclic.

Counterexample

A Counterexample is an example that demonstrates that a statement is not true.

Examples:

- **Statement**: If $x^2 = 4$ then $x = 2$.

  Counterexample: x = –2 provides a counterexample.

  Statement: If the diagonals of a quadrilateral intersect at right angles, then the quadrilateral is a rhombus.

  Counterexample: A kite with the diagonals not bisecting each other is not a rhombus. Such a kite provides a counterexample to the statement. The diagonals of a kite do intersect at right angles.

  Statement: Every convex quadrilateral is a cyclic quadrilateral.

  Counterexample: A parallelogram that is not a rectangle is convex, but not cyclic.

Equivalent statements

Statements P and Q are equivalent if P $\Rightarrow$ Q and Q $\Rightarrow$ P. The symbol $\Leftrightarrow$ is used. It is also written as P if and only if Q or P iff Q.

Examples:

- A quadrilateral is a rectangle if and only if the diagonals of the quadrilateral are of equal length and bisect each other.
- A quadrilateral is cyclic if and only if opposite angles are supplementary.

Negation

If P is a statement, then the statement 'not P', denoted by $\neg$P is the negation of P. If P is the statement 'It is snowing.' then $\neg$P is the statement 'It is not snowing.'.

Quantifiers

For all (For each)

Symbol ∀

- For all real numbers x, $x^2 \geq 0$. (∀ real numbers x, $x^2 \geq 0$.)
- For all triangles the sum of the interior angles is 180 .(∀ triangles the sum of the interior angles is 180°.)
- For each diameter of a given circle each angle subtended at the circumference by that diameter is a right angle.

There exists

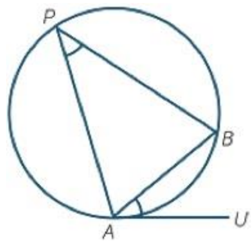
Symbol ∃

- There exists a real number that is not positive (∃ a real number that is not positive.)
- There exists a prime number that is not odd. (∃ a prime number that is not odd.)
- There exists a natural number that is less than 6 and greater than 3.
- There exists an isosceles triangle that is not equilateral.

The quantifiers can be used together. For example: $\forall\ x \geq 0, \exists\ y \geq 0$ such that $y^2 = x$.

Glossary of Geometric Terms and Listing of Important Theorems

Alternate segment

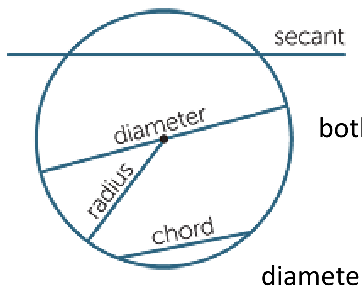


The word 'alternate' means 'other'. The chord *AB* divides the circle into two segments and *AU* is tangent to the circle. Angle *APB* 'lies in' the segment on the other side of chord *AB* from angle *BAU.* We say that it is in the **alternate segment**.

Cyclic quadrilateral

A **cyclic quadrilateral** is a quadrilateral whose vertices all lie on a circle.

Lines and line segments associated with circles



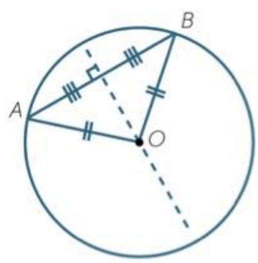
Any line segment joining a point on the circle to the centre is called a **radius**. By the definition of a circle, any two radii have the same length called the radius of the circle. Notice that the word 'radius' is used to refer both to these intervals and to the common length of these intervals.

- An interval joining two points on the circle is called a **chord**.
- A chord that passes through the centre is called a **diameter**. Since a diameter consists of two radii joined at their endpoints, every diameter has length equal to twice the radius. The word 'diameter' is used to refer both to these intervals and to their common length.

A line that cuts a circle at two distinct points is called a **secan**t. Thus a chord is the interval that the circle cuts off a secant, and a diameter is the interval cut off by a secant passing through the centre of a circle.

Circle Theorems

Result 1

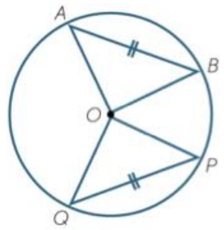


Let *AB* be a chord of a circle with centre *O*.

The following three lines coincide:

- The bisector of the angle ∠*AOB* subtended at the centre by the chord.
- The line segment (interval) joining *O* and the midpoint of the chord *AB*.
- The perpendicular bisector of the chord *AB*.

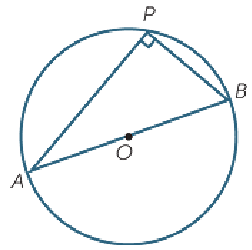
Result 2



- Equal chords of a circle subtend equal angles at the centre.

In the diagram shown ∠*AOB* = ∠*POQ*.

Result 3



- An angle in a semicircle is a right angle.

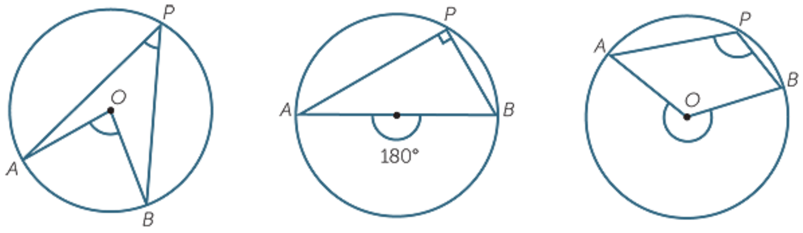
Let *AOB* be a diameter of a circle with centre *O*, and let *P* be any other point on the circle. The angle ∠*APB* subtended at *P* by the diameter *AB* is called an **angle in a semicircle**.

Converse

- The circle whose diameter is the hypotenuse of a right-angled triangle passes through all three vertices of the triangle.

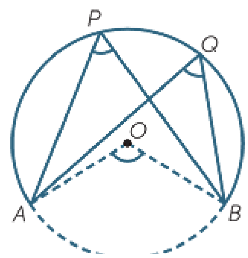
Result 4

- An angle at the circumference of a circle is half the angle subtended at the centre by the same arc. In the diagram shown ∠*AOB* = 2∠*APB*



The arc *AB* **subtends** the angle ∠*AOB* at the centre. The arc also subtends the angle ∠*APB*, called an **angle at the circumference** subtended by the arc *AB*.

Result 5



- Two angles at the circumference subtended by the same arc are equal.

∠APB = ∠AQB

In the diagram, the two angles ∠*APB* and ∠*AQB* are subtended by the same arc *AB*.

Result 6

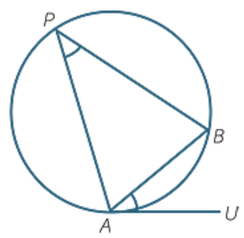
- The opposite angles of a cyclic quadrilateral are supplementary.

Converse

- If the opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.

Result 7

Alternate segment theorem



An angle between a chord and a tangent is equal to any angle in the alternate segment.

In the diagram ∠*BAU* = ∠*APB.*

Unit 2

Trigonometry

Angle sum and difference identities

$\sin(A + B) = \sin A \cos B + \sin B \cos A$

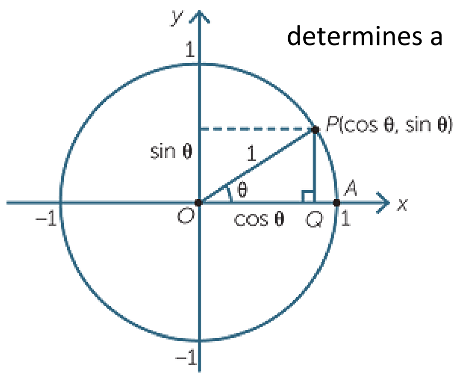
$\sin(A - B) = \sin A \cos B - \sin B \cos A$

$\cos(A + B) = \cos A \cos B - \sin A \sin B$

$\cos(A - B) = \cos A \cos B + \sin A \sin B$

Cosine and sine functions

Since each angle θ measured anticlockwise from the positive *x*-axis determines a point P on the unit circle, we will define



- the cosine of θ to be the x-coordinate of the point *P*
- the sine of θ to be the y-coordinate of the point *P*
- the tangent of θ is the gradient of the line segment *OP*

Double angle formula

- $\sin 2A = 2 \sin A \cos A$
- $\cos 2A = \cos^2 A - \sin^2 A$
  $= 2\cos^2 A - 1$
  $= 1 - 2\sin^2 A$
- $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

Products as sums and differences

- $\cos A \cos B = \frac{1}{2}(\cos(A - B) + \cos(A + B))$
- $\sin A \sin B = \frac{1}{2}(\cos(A - B) - \cos(A + B))$
- $\sin A \cos B = \frac{1}{2}(\sin(A + B) + \sin(A - B))$
- $\cos A \sin B = \frac{1}{2}(\sin(A + B) - \sin(A - B))$

Pythagorean identities

- $\cos^2 A + \sin^2 A = 1$
- $\tan^2 A + 1 = \sec^2 A$
- $\cot^2 A + 1 = \operatorname{cosec}^2 A$

Reciprocal trigonometric functions

- $\sec A = \dfrac{1}{\cos A}$, $\cos A \neq 0$
- $\operatorname{cosec} A = \dfrac{1}{\sin A}$, $\sin A \neq 0$
- $\cot A = \dfrac{\cos A}{\sin A}$, $\sin A \neq 0$

Matrices

Addition of matrices (See Matrix)

If ***A*** and ***B*** are matrices with the same dimensions and the entries of ***A*** are a_{ij} and the entries of ***B*** are b_{ij} then the entries of ***A* + *B*** are $a_{ij} + b_{ij}$

For example if ***A*** = $\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and ***B*** = $\begin{bmatrix} 5 & 1 \\ 2 & 1 \\ 1 & 6 \end{bmatrix}$ then

$$\boldsymbol{A} + \boldsymbol{B} = \begin{bmatrix} 7 & 2 \\ 2 & 4 \\ 2 & 10 \end{bmatrix}$$

Determinant of a 2 × 2 matrix (See Matrix)

If ***A*** = $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ the determinant of ***A*** denoted as det ***A*** = $ad - bc$.

If det ***A*** ≠ 0,

- the matrix ***A*** has an **inverse**.
- the simultaneous linear equations $ax + by = e$ and $cx + dy = f$ have a unique solution.
- The linear transformation of the plane, defined by ***A*** maps the unit square

  $O(0, 0)$, $B(0,1)$, $C(1, 1)$, $D(1, 0)$ to a parallelogram $OB'C'D'$ of area |det ***A*** |.
- The sign of the determinant determines the orientation of the image of a figure under the transformation defined by the matrix.

Dimension (or size) (See Matrix)

Two matrices are said to have the same **dimensions** (or **size**) **if** they have the same number of rows and columns.

- For example, the matrices

  $\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ and $\begin{bmatrix} 3 & 4 & 5 \\ 6 & 7 & 8 \end{bmatrix}$

have the same dimensions. They are both 2 × 3 matrices.

- An $m \times n$ matrix has m rows and n columns.

Entries (Elements) of a matrix

The symbol a_{ij} represents the (i, j) entry which occurs in the i^{th} row and the j^{th} column. For example a general 3×2 matrix is:

$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and a_{32} is the entry in the third row and the second column.

Leading diagonal

The leading diagonal of a square matrix is the diagonal which runs from the top left corner to the bottom right corner.

Linear transformation defined by a 2 × 2 matrix

The matrix multiplication

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax + by \\ cx + dy \end{bmatrix}$$

defines a transformation $T(x, y) = (ax + by, cx + dy)$.

Linear transformations in 2 dimensions

A linear transformation in the plane is a mapping of the form

$T(x, y) = (ax + by, cx + dy)$.

A transformation T is linear if and only if

$T(\alpha(x_1, y_1) + \beta(x_2, y_2) = \alpha T(x_1, y_1) + \beta T(x_2, y_2)$.

Linear transformations include:

- rotations around the origin
- reflections in lines through the origin
- dilations.

Translations are not linear transformations.

Matrix (matrices)

A **matrix** is a rectangular array of elements or entries displayed in rows and columns.

For example,

$\boldsymbol{A} = \begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix}$ and $\boldsymbol{B} = \begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix}$ are both matrices.

Matrix $\boldsymbol{A}$ is said to be a 3×2 matrix (three rows and two columns) while $\boldsymbol{B}$ is said to be a 2×3 matrix (two rows and three columns).

A **square matrix** has the same number of rows and columns.

A **column matrix** (or vector) has only one column.

A **row matrix** (or vector) has only one row.

Matrix algebra of 2 × 2 matrices

If ***A, B*** and ***C*** are 2 × 2 matrices, ***I*** the 2 × 2 (multiplicative) identity matrix and ***O*** the 2 × 2 zero matrix then:

$A + B = B + A$	(commutative law for addition)
$(A + B) + C = A + (B + C)$	(associative law for addition)
$A + O = A$	(additive identity)
$A + (-A) = O$	(additive inverse)
$(AB)C = A(BC)$	(associative law for multiplication)
$AI = A = IA$	(multiplicative identity)
$A(B + C) = AB + AC$	(left distributive law)
$(B + C)A = BA + CA$	(right distributive law)

Matrix multiplication

Matrix multiplication is the process of multiplying a matrix by another matrix. The product ***AB*** of two matrices ***A*** and ***B*** with **dimensions** $m \times n$ and $p \times q$ is defined if $n = p$. If it is defined, the product ***AB*** is an $m \times q$ matrix and it is computed as shown in the following example.

$$\begin{bmatrix} 1 & 8 & 0 \\ 2 & 5 & 7 \end{bmatrix} \begin{bmatrix} 6 & 10 \\ 11 & 3 \\ 12 & 4 \end{bmatrix} = \begin{bmatrix} 94 & 34 \\ 151 & 63 \end{bmatrix}$$

The entries are computed as shown

$1 \times 6 + 8 \times 11 + 0 \times 12 = 94$

$1 \times 10 + 8 \times 3 + 0 \times 4 = 34$

$2 \times 6 + 5 \times 11 + 7 \times 12 = 151$

$2 \times 10 + 5 \times 3 + 7 \times 4 = 63$

The entry in row i and column j of the product ***AB*** is computed by 'multiplying' row i of ***A*** by column j of **B** as shown.

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$ then

$$AB = \begin{bmatrix} a_{11}b_{11} + a_{12}b_{21} & a_{11}b_{12} + a_{12}b_{22} & a_{11}b_{13} + a_{12}b_{23} \\ a_{21}b_{11} + a_{22}b_{21} & a_{21}b_{12} + a_{22}b_{22} & a_{21}b_{13} + a_{22}b_{23} \\ a_{31}b_{11} + a_{32}b_{21} & a_{31}b_{12} + a_{32}b_{22} & a_{31}b_{13} + a_{32}b_{23} \end{bmatrix}$$

(Multiplicative) identity matrix

A **(multiplicative) identity matrix** is a square matrix in which all the elements in the leading diagonal are 1s and the remaining elements are 0s. Identity matrices are designated by the letter ***I*.**

For example,

$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ and $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ are both identity matrices.

There is an identity matrix for each order of square matrix. When clarity is needed, the order is written with a subscript: $\boldsymbol{I}_n$

Multiplicative inverse of a square matrix

The inverse of a square matrix $\boldsymbol{A}$ is written as $\boldsymbol{A}^{-1}$ and has the property that

$$\boldsymbol{A}\boldsymbol{A}^{-1} = \boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{I}$$

Not all square matrices have an inverse. A matrix that has an inverse is said to be **invertible**.

Multiplicative inverse of a 2 × 2 matrix

The **inverse** of the matrix $\boldsymbol{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is $\boldsymbol{A}^{-1} = \dfrac{1}{\det \boldsymbol{A}}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$, when $\det \boldsymbol{A} \neq 0$.

Scalar multiplication (matrices)

Scalar multiplication is the process of multiplying a matrix by a scalar (number).

For example, forming the product:

$$10\begin{bmatrix} 2 & 1 \\ 0 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 20 & 10 \\ 0 & 30 \\ 10 & 40 \end{bmatrix}$$

is an example of the process of scalar multiplication.

In general for the matrix $\boldsymbol{A}$ with entries a_{ij} the entries of $k\boldsymbol{A}$ are ka_{ij}.

Singular matrix

A matrix is singular if $\det \boldsymbol{A} = 0$. A singular matrix does not have a multiplicative inverse.

Zero matrix

A matrix is a zero matrix if all of its entries are zero. For example:

$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ are zero matrices.

There is a zero matrix for each **size** of matrix. When clarity is needed, we write $\mathbf{O}_{n \times m}$ for the $n \times m$ zero matrix.

Real and Complex Numbers

Complex numbers

Complex arithmetic

If $z_1 = x_1 + y_1 i$ and $z_2 = x_2 + y_2 i$

- $z_1 + z_2 = x_1 + x_2 + (y_1 + y_2)\, i$
- $z_1 - z_2 = x_1 - x_2 + (y_1 - y_2)\, i$
- $z_1 \times z_2 = x_1 x_2 - y_1 y_2 + (x_1 y_2 + x_2 y_1)i$
- $z_1 \times (0 + 0i) = 0$ Note: $0 + 0i$ is usually written as 0
- $z_1 \times (1 + 0i) = z_1$ Note: $1 + 0i$ is usually written as 1

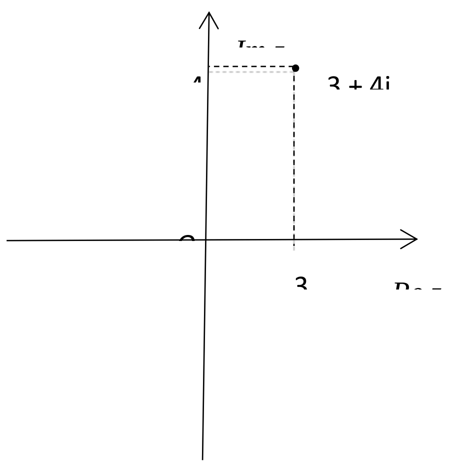
Complex conjugate

For any complex number $z = x + iy$, its **conjugate** is $\bar{z} = x - iy$. The following properties hold

- $\overline{z_1 z_2} = \bar{z}_1\, \bar{z}_2$
- $\overline{z_1/z_2} = \bar{z}_1 / \bar{z}_2$
- $z\,\bar{z} = |z|^2$
- $z + \bar{z}$ is real

Complex plane (Argand plane)

The **complex plane** is a geometric representation of the complex numbers established by the **real axis** and the orthogonal **imaginary axis**. The complex plane is sometimes called the Argand plane.



Imaginary part of a complex number

A complex number z may be written as $x + yi$, where x and y are real, and then y is the imaginary part of z. It is denoted by $Im\ (z)$.

Integers

The **integers** are the numbers $\cdots, -3, -2, -1, 0, 1, 2, 3, \ldots$

Modulus (Absolute value) of a complex number

If z is a complex number and $z = x + iy$ then the modulus of z is the distance of z from the origin in the Argand plane. The modulus of z denoted by $|z| = \sqrt{x^2 + y^2}$.

Prime numbers

A prime number is a positive integer greater than 1 that has no positive integer factor other 1 and itself. The first few prime numbers are $2, 3, 5, 7, 11, 13, 17, 19, 23 \ldots$

Principle of mathematical induction

Let there be associated with each positive integer n, a proposition $P(n)$.

If

1. $P(1)$ is true, and
2. for all k, $P(k)$ is true implies $P(k + 1)$ is true,

then $P(n)$ is true for all positive integers n.

Rational numbers

A real number is **rational** if it can be expressed as a quotient of two integers. Otherwise it is called irrational.

Irrational numbers can be approximated as closely as desired by rational numbers, and most electronic calculators use a rational approximation when performing calculations involving an irrational number.

Real numbers

The numbers generally used in mathematics, in scientific work and in everyday life are the **real numbers**. They can be pictured as points on a number line, with the integers evenly spaced along the line, and a real number a to the right of a real number b if $a > b$.

A real number is either rational or irrational. The set of real numbers consists of the set of all rational and irrational numbers.

Every real number has a decimal expansion. Rational numbers are the ones whose decimal expansions are either terminating or eventually recurring.

Real part of a complex number

A complex number z may be written as $x + yi$, where x and y are real, and then x is the real part of z. It is denoted by $Re(z)$.

Whole numbers

A **whole number** is a non–negative integer, that is, one of the numbers 0, 1, 2, 3, ...

Unit 3

Complex Numbers

Argument (abbreviated arg)

If a complex number is represented by a point P in the complex plane, then the argument of z, denoted arg z, is the angle θ that OP makes with the positive real axis O_x, with the angle measured anticlockwise from O_x. The **principal value** of the argument is the one in the interval $(-\pi, \pi]$.

Complex arithmetic (see Complex arithmetic Unit 2)

Complex conjugate (see Complex conjugate Unit 2)

De Moivre's Theorem

For all integers n, $(\cos\theta + i\sin\theta)^n = \cos n\theta + i\sin n\theta$.

Modulus of a complex number (Modulus of a complex number Unit 2)

Polar form of a complex number

For a complex number z, let $\theta = \arg z$. Then $z = r(\cos\theta + i\sin\theta)$ is the polar form of z.

Root of unity (nth root of unity)

A complex number z such that $z^n = 1$

The n^{th} roots of unity are:

$$\cos\frac{2k\pi}{n} + i\sin\frac{2k\pi}{n} \qquad \text{where } k = 0, 1, 2, \ldots, n-1.$$

The points in the complex plane representing roots of unity lie on the unit circle.

The cube roots of unity are

$z_1 = 1$, $z_2 = \frac{1}{2}(-1 + i\sqrt{3})$, $z_3 = \frac{1}{2}(-1 - i\sqrt{3})$. Note $z_3 = \overline{z_2}$ and $z_3 = \frac{1}{z_2}$ and $z_2 z_3 = 1$.

Functions and sketching graphs

Rational function

A rational function is a function such that $f(x) = \frac{g(x)}{h(x)}$, where $g(x)$ and $h(x)$ are polynomials. Usually $g(x)$ and $h(x)$ are chosen so as to have no common factor of degree greater than or equal to 1, and the domain of f is usually taken to be $R\backslash\{x: h(x) = 0\}$.

Vectors in three-dimensions

(See Vectors in Unit 2)

Addition of vectors

In component form if $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$\mathbf{a} + \mathbf{b} = (a_1 + b_1)\mathbf{i} + (a_2 + b_2)\mathbf{j} + (a_3 + b_3)\mathbf{k}$

Vector equation of a straight line

Let **a** be the position vector of a point on a line *l*, and **u** any vector with direction along the line. The line consists of all points *P* whose position vector **p** is given by

$\mathbf{p} = \mathbf{a} + t\mathbf{u}$ for some real number t.

(Given the position vectors of two points on the plane **a** and **b** the equation can be written as

$\mathbf{p} = \mathbf{a} + t(\mathbf{b} - \mathbf{a})$ for some real number t.)

Vector equation of a plane

Let **a** be a position vector of a point *A* in the plane, and **n** a normal vector to the plane . Then the plane consists of all points *P* whose position vector **p** satisfies

$(\mathbf{p} - \mathbf{a}).\mathbf{n} = \mathbf{0}$. This equation may also be written as $\mathbf{p}.\mathbf{n} = \mathbf{a}.\mathbf{n}$, a constant.

(If the the normal vector **n** is the vector (l, m, n) in ordered triple notation and the scalar product

$\mathbf{a}.\mathbf{n} = \mathrm{k}$, this gives the Cartesian equation $lx + my + nz = k$ for the plane)

Vector function

In this course a vector function is one that depends on a single real number parameter t, often representing time, producing a vector $\boldsymbol{r}(t)$ as the result. In terms of the standard unit vectors **i**, **j**, **k** of three dimensional space, the vector-valued functions of this specific type are given by expressions such as

$\boldsymbol{r}(t) = f(t)\mathbf{i} + g(t)\mathbf{j} + h(t)\mathbf{k}$

where f, g and h are real valued functions giving coordinates.

Scalar product

If $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ then the scalar product $\mathbf{a}.\mathbf{b}$ is the real number

$a_1b_1 + a_2b_2 + a_3b_3$.

When expressed in **i**, **j**, **k** notation, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$

Vector product (Cross product)

When expressed in **i**, **j**, **k** notation, $\mathbf{a} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ and $\mathbf{b} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ then

$\mathbf{a} \times \mathbf{b} = (a_2b_3 - a_3b_2)\mathbf{i} + (a_3b_1 - a_1b_3)\mathbf{j} + (a_1b_2 - a_2b_1)\mathbf{k}$

The cross product has the following geometric interpretation. Let **a** and **b** be two non- parallel vectors then $|\mathbf{a} \times \mathbf{b}|$ is the area of the parallelogram defined by **a** and **b** and

$\mathbf{a} \times \mathbf{b}$ is normal to this parallelogram.

(The cross product of two parallel vectors is the zero vector.)

Unit 4

Integration and applications of integration

Inverse Trigonometric functions

The inverse sine function, $y = \sin^{-1} x$

If the domain for the sine function is restricted to the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ a one to one function is formed and so an inverse function exists.

The inverse of this restricted sine function is denoted by $\sin^{-1}$ and is defined by:

$$\sin^{-1}: [-1, 1] \to R,\ \sin^{-1} x = y \text{ where } \sin y = x,\ y \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$\sin^{-1}$ is also denoted by arcsin.

The inverse cosine function, $y = \cos^{-1} x$

If the domain of the cosine function is restricted to $[0, \pi]$ a one to one function is formed and so the inverse function exists.

$\cos^{-1} x$, the inverse function of this restricted cosine function, is defined as follows:

$$\cos^{-1}: [-1, 1] \to R,\ \cos^{-1} x = y \text{ where } \cos y = x,\ y \in [0, \pi]$$

$\cos^{-1}$ is also denoted by arccos.

The inverse tangent function, $y = \tan^{-1} x$

If the domain of the tangent function is restricted to $(-\frac{\pi}{2}, \frac{\pi}{2})$ a one to one function is formed and so the inverse function exists.

$$\tan^{-1}: R \to R,\ \tan^{-1} x = y \text{ where } \tan y = x,\ y \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

$\tan^{-1}$ is also denoted by arctan.

Rates of change and differential equations

Implicit differentiation

When variables x and y satisfy a single equation, this may define y as a function of x even though there is no explicit formula for y in terms of x. **Implicit differentiation** consists of differentiating each term of the equation as it stands and making use of the chain rule. This can lead to a formula for $\frac{dy}{dx}$. For example,

if $x^2 + xy^3 - 2x + 3y = 0$,

then $2x + x(3y^2)\frac{dy}{dx} + y^3 - 2 + 3\frac{dy}{dx} = 0$,

and so $\frac{dy}{dx} = \frac{2-2x-y^3}{3xy^2+3}$.

Linear momentum

The linear momentum **p** of a particle is the vector quantity $\mathbf{p} = m\mathbf{v}$ where m is the mass and $\mathbf{v}$ is the velocity.

Logistic equation

The logistic equation has applications in a range of fields, including biology, biomathematics, economics, chemistry, mathematical psychology, probability, and statistics.

One form of this differential equation is:

$$\frac{dy}{dt} = ay - by^2 \qquad \text{(where } a > 0 \text{ and } b > 0\text{)}$$

The general solution of this is

$y = \frac{a}{b + Ce^{-at}}$, where C is an arbitrary constant.

Separation of variables

Differential equations of the form $\frac{dy}{dx} = g(x)h(y)$ can be rearranged as long as $h(y) \neq 0$ to obtain

$\frac{1}{h(y)}\frac{dy}{dx} = g(x)$.

Slope field

Slope field (**direction or gradient field**) is a graphical representation of the solutions of a linear first-order differential equation in which the derivative at a given point is represented by a line segment of the corresponding slope

Statistical Inference

Continuous random variable

A random variable X is called continuous if its set of possible values consists of intervals, and the chance that it takes any point value is zero (in symbols, if $P(X = x) = 0$ for every real number x). A random variable is continuous if and only if its cumulative probability distribution function can be expressed as an integral of a function.

Probability density function

The probability density function (pdf) of a continuous random variable is the function that when integrated over an interval gives the probability that the continuous random variable having that pdf, lies in that interval. The **probability density function** is therefore the derivative of the (cumulative probability) distribution function.

Precision

Precision is a measure of how close an estimator is expected to be to the true value of the parameter it purports to estimate.

Independent and identically distributed observations

For independent observations, the value of any one observation has no effect on the chance of values for all the other observations. For identically distributed observations, the chances of the possible values of each observation are governed by the same probability distribution.

Random sample

A random sample is a set of data in which the value of each observation is governed by some chance mechanism that depends on the situation. The most common situation in which the term "random sample" is used refers to a set of independent and identically distributed observations.

Samples mean the arithmetic average of the sample values

Appendix G – Course Adoption

Conditions of Adoption

The course and units of this course are consistent with the philosophy and goals of the college and the adopting college has the human and physical resources to implement the course.

Adoption Process

Course adoption must be initiated electronically by an email from the principal or their nominated delegate to bssscertification@ed.act.edu.au. A nominated delegate must CC the principal.

The email will include the **Conditions of Adoption** statement above, and the table below adding the **College** name, and circling the **Classification/s** required.

College:	
Course Title:	Specialist Mathematics
Classification/s:	T
Accredited from:	2014
Framework:	Mathematics 2020