



ANU H Course Specialist Mathematics



Written under the Mathematics Framework
Accredited from 2021 – 2023



Australian
National
University

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Fiona Foley, Winged harvest 2001, Wood, aluminium, ochre, and stainless steel, commissioned 2000
(WEH Stanner Building courtyard)

Table of Contents

H Courses	1
The ACT Senior Secondary System	1
ACT Senior Secondary Certificate	2
Learning Principles	3
General Capabilities	4
Cross-Curriculum Priorities	6
ANU H Course Specialist Mathematics	7
Rationale	7
Goals	7
Organisation of Content	8
Unit Titles	8
Assessment	8
Achievement Standards	10
Cryptography and Infinite Cardinals	Value: 1.0 13
Chaos, Fractals; Geometry, Topology	Value: 1.0 15
Appendix A – Implementation Guidelines	17
Appendix B – Course Developers	20
Appendix C – Common Curriculum Elements	21
Appendix D – Glossary of Verbs	22
Appendix E – Glossary for ACT Senior Secondary Curriculum	23

H Courses

H classification is given to a year 11 and 12 course which is designed and accredited by the Board of Senior Secondary Studies (BSSS) and an Australian university, and where successful completion of the course will be recognised both towards the ACT Senior Secondary Certificate and an undergraduate degree with that university.

The BSSS considers H courses as complementary to studies in the home college. These extension courses allow students to pursue depth of study in an area of interest, while also gaining experience in a tertiary context to prepare for future studies.

The ACT Senior Secondary System

The ACT senior secondary system recognises a range of university, vocational or life skills pathways.

The system is based on the premise that teachers are experts in their area: they know their students and community and are thus best placed to develop curriculum and assess students according to their needs and interests. Students have ownership of their learning and are respected as young adults who have a voice.

A defining feature of the system is school-based curriculum and continuous assessment. School-based curriculum provides flexibility for teachers to address students' needs and interests. College teachers have an opportunity to develop courses for implementation across ACT schools. Based on the courses that have been accredited by the BSSS, college teachers are responsible for developing programs of learning. A program of learning is developed by individual colleges to implement the courses and units they are delivering.

Teachers must deliver all content descriptions; however, they do have flexibility to emphasise some content descriptions over others. It is at the discretion of the teacher to select the texts or materials to demonstrate the content descriptions. Teachers can choose to deliver course units in any order and teach additional (not listed) content provided it meets the specific unit goals.

School-based continuous assessment means that students are continually assessed throughout years 11 and 12, with both years contributing equally to senior secondary certification. Teachers and students are positioned to have ownership of senior secondary assessment. The system allows teachers to learn from each other and to refine their judgement and develop expertise.

Senior secondary teachers have the flexibility to assess students in a variety of ways. For example: multimedia presentation, inquiry-based project, test, essay, performance and/or practical demonstration may all have their place. College teachers are responsible for developing assessment instruments with task specific rubrics and providing feedback to students.

The integrity of the ACT Senior Secondary Certificate is upheld by a robust, collaborative, and rigorous structured consensus-based peer reviewed moderation process. System moderation involves all year 11 and 12 teachers from public, non-government and international colleges delivering the ACT Senior Secondary Certificate.

Only students who desire a pathway to university are required to sit a general aptitude test, referred to as the ACT Scaling Test (AST), which moderates student scores across courses and colleges. Students are required to use critical and creative thinking skills across a range of disciplines to solve problems. They are also required to interpret a stimulus and write an extended response.

Senior secondary curriculum makes provision for student-centred teaching approaches, integrated and project-based learning inquiry, formative assessment, and teacher autonomy. ACT Senior Secondary Curriculum makes provision for diverse learners and students with mild to moderate intellectual disabilities, so that all students can achieve an ACT Senior Secondary Certificate.

The ACT Board of Senior Secondary Studies (BSSS) leads senior secondary education. It is responsible for quality assurance in senior secondary curriculum, assessment, and certification. The Board consists of nominees from colleges, professional bodies, universities, industry, parent/carer organisations and unions. The Office of the Board of Senior Secondary Studies (OBSSS) consists of professional and administrative staff who support the Board in achieving its objectives and functions.

ACT Senior Secondary Certificate

Courses of study for the ACT Senior Secondary Certificate:

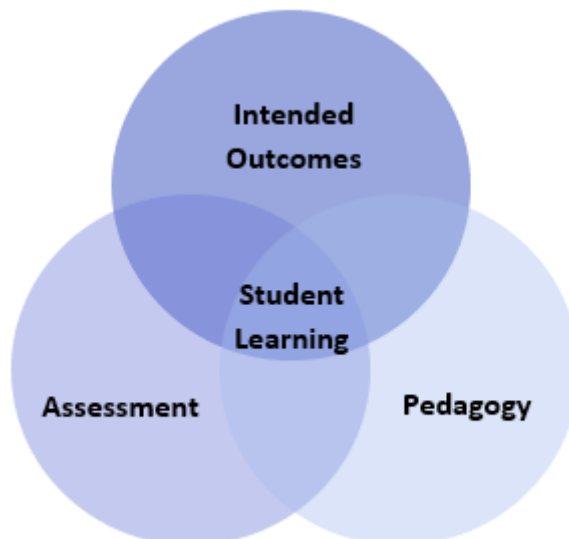
- provide a variety of pathways, to meet different learning needs and encourage students to complete their secondary education
- enable students to develop the essential capabilities for twenty-first century learners
- empower students as active participants in their own learning
- engage students in contemporary issues relevant to their lives
- foster students' intellectual, social, and ethical development
- nurture students' wellbeing, and physical and spiritual development
- enable effective and respectful participation in a diverse society.

Each course of study:

- comprises an integrated and interconnected set of knowledge, skills, behaviours, and dispositions that students develop and use in their learning across the curriculum
- is based on a model of learning that integrates intended student outcomes, pedagogy, and assessment
- outlines teaching strategies which are grounded in learning principles and encompass quality teaching
- promotes intellectual quality, establish a rich learning environment, and generate relevant connections between learning and life experiences
- provides formal assessment and certification of students' achievements.

Underpinning beliefs

- All students are able to learn.
- Learning is a partnership between students and teachers.
- Teachers are responsible for advancing student learning.



Learning Principles

1. Learning builds on existing knowledge, understandings, and skills.
(*Prior knowledge*)
2. When learning is organised around major concepts, principles, and significant real world issues, within and across disciplines, it helps students make connections and build knowledge structures.
(*Deep knowledge and connectedness*)
3. Learning is facilitated when students actively monitor their own learning and consciously develop ways of organising and applying knowledge within and across contexts.
(*Metacognition*)
4. Learners' sense of self and motivation to learn affects learning.
(*Self-concept*)
5. Learning needs to take place in a context of high expectations.
(*High expectations*)
6. Learners learn in different ways and at different rates.
(*Individual differences*)
7. Different cultural environments, including the use of language, shape learners' understandings and the way they learn.
(*Socio-cultural effects*)
8. Learning is a social and collaborative function as well as an individual one.
(*Collaborative learning*)
9. Learning is strengthened when learning outcomes and criteria for judging learning are made explicit and when students receive frequent feedback on their progress.
(*Explicit expectations and feedback*)

General Capabilities

All courses of study for the ACT Senior Secondary Certificate should enable students to develop essential capabilities for twenty-first century learners. These ‘capabilities’ comprise an integrated and interconnected set of knowledge, skills, behaviours and dispositions that students develop and use in their learning across the curriculum.

The capabilities include:

- literacy
- numeracy
- information and communication technology (ICT)
- critical and creative thinking
- personal and social
- ethical understanding
- intercultural understanding

Courses of study for the ACT Senior Secondary Certificate should be both relevant to the lives of students and incorporate the contemporary issues they face. Hence, courses address the following three priorities. These priorities are:

- Aboriginal and Torres Strait Islander histories and cultures
- Asia and Australia’s engagement with Asia
- Sustainability

Elaboration of these General Capabilities and priorities is available on the ACARA website at www.australiancurriculum.edu.au.

Literacy

In the *ANU H Course Specialist Mathematics* literacy skills and strategies for accessing data at the tertiary level to enable students to express, interpret, and communicate complex mathematical information, ideas, and processes. Mathematics provides a specific and rich context for students to develop their ability to read, write, visualise, and talk about complex situations involving a range of mathematical ideas. Students can apply and further develop their literacy skills and strategies by shifting between verbal, graphic, numerical, and symbolic forms of representing problems in order to formulate, understand and solve problems and communicate results. This process of translation across different systems of representation is essential for complex mathematical reasoning and expression. Students learn to communicate their findings in different ways, using multiple systems of representation and data displays to illustrate the relationships they have observed or constructed.

Numeracy

The students who undertake this subject will continue to develop their numeracy skills at a more sophisticated level than in Years 11-12. They will extend their conceptualisation of numeracy into the tertiary level. This subject contains topics that will equip students for the ever-increasing demands of the information age.

Information and Communication Technology (ICT) Capability

In the *ANU H Course Specialist Mathematics*, students use ICT both to develop theoretical mathematical understanding and to apply mathematical knowledge to a range of problems. With access to ANU facilities and expertise, they use software aligned with areas of work and society with which they may be involved such as for statistical analysis, algorithm generation, and manipulation, and complex calculation. They use digital tools to make connections between mathematical theory, practice, and application; for example, to use data, to address problems, and to operate systems in authentic situations.

Critical and Creative Thinking

In the *ANU H Course Specialist Mathematics*, students operate at a level of Mathematical creativity and critical theory comparable to tertiary courses. They compare predictions with observations when evaluating a theory. They check the extent to which their theory-based predictions match observations. They assess whether, if observations and predictions don't match, it is due to a flaw in theory or method of applying the theory to make predictions – or both. They revise, or reapply their theory more skilfully, recognising the importance of self-correction in the building of useful and accurate theories and making accurate predictions.

Personal and Social Capability

In the *ANU H Course Specialist Mathematics* students enter into the tertiary environment and gain experience with the social and personal skills that will be helpful to success at university. They develop personal and social competence in working with others to address problems in Mathematics. They gain experience through setting and monitoring personal and academic goals, taking initiative, building adaptability, communication, teamwork, and decision-making.

Ethical Understanding

In the *ANU H Course Specialist Mathematics*, students develop ethical understanding in Mathematics through decision-making connected with ethical dilemmas that arise when engaged in mathematical calculation and the dissemination of results and the social responsibility associated with teamwork and attribution of input.

The areas relevant to Mathematics include issues associated with ethical decision-making as students work collaboratively in teams and independently as part of their mathematical explorations and investigations. Acknowledging errors rather than denying findings and/or evidence involves resilience and examined ethical understanding. They develop increasingly advanced communication, research, and presentation skills to express viewpoints.

Intercultural Understanding

In the *ANU H Course Specialist Mathematics*, students understand Mathematics as a socially constructed body of knowledge that uses universal symbols but has its origin in many cultures. Students understand that some languages make it easier to acquire mathematical knowledge than others. Students also understand that there are many culturally diverse forms of mathematical knowledge, including diverse relationships to number and that diverse cultural spatial abilities and understandings are shaped by a person's environment and language.

Cross-Curriculum Priorities

Aboriginal and Torres Strait Islander Histories and Cultures

The *ANU H Course Specialist Mathematics* curriculum values the histories, cultures, traditions and languages of Aboriginal and Torres Strait Islander Peoples past and ongoing contributions to contemporary Australian society and culture. Through the study of Mathematics within relevant contexts, opportunities will allow for the development of students' understanding and appreciation of the diversity of Aboriginal and Torres Strait Islander Peoples histories and cultures.

Asia and Australia's Engagement with Asia

There are strong social, cultural, and economic reasons for Australian students to engage with the countries of Asia and with the past and ongoing contributions made by the peoples of Asia in Australia. It is through the study of Mathematics in an Asian context that students engage with Australia's place in the region. Through analysis of relevant data, students are provided with opportunities to further develop an understanding of the diverse nature of Asia's environments and traditional and contemporary cultures.

Sustainability

In the *ANU H Course Specialist Mathematics* provides the opportunity for the development of informed and reasoned points of view, discussion of issues, research and problem solving. Therefore, teachers are encouraged to select contexts for discussion connected with sustainability. Through analysis of data, students have the opportunity to research and discuss this global issue and learn the importance of respecting and valuing a wide range of world perspectives.

ANU H Course

Specialist Mathematics

Rationale

The goal is to introduce students to contemporary mainstream 20th and 21st century Mathematics. Students will investigate some very exciting and useful modern mathematics and get a feeling for "what mathematics is all about".

The Mathematics which students will see in this course is usually not covered until higher level courses in second or third year at University. Naturally, it is then studied in much greater depth. Students will study carefully chosen parts and representative examples from various areas of mathematics which illustrate important and general key concepts. In the process students will have the opportunity to gain a real understanding and feeling for the beauty, utility, and breadth of mathematics.

Students will build their capacity as students by developing planning, communication, and collaboration skills necessary to complete Mathematical investigations in a tertiary environment. They will develop as people by interacting with instructors and students from a wide range of schools and back grounds. They will learn to interact with others in a productive tertiary environment, to better prepare them for further studies.

Goals

All courses based on the Mathematics Framework should enable students to:

- critically and creatively apply mathematical concepts, models, and techniques
- evaluate the reasonableness of solutions to problems
- develop a critical appreciation of the use of information and communication technology in mathematics
- communicate using appropriate mathematical language
- develop mathematical judgements and arguments through inquiry
- learn with purpose and persistence, independently and collaboratively
- evaluate the potential of mathematics to generate knowledge in the public good
- reflect on thinking and learning.

Unit Titles

- Cryptography and Infinite Cardinals
- Chaos, Fractals; Geometry, Topology

Organisation of Content

Cryptography and Infinite Cardinals

In this unit, students use a number of techniques to solve mathematical problems. They demonstrate an understanding of mathematical patterns and their application to cryptography. They develop an understanding of real number systems, including concepts such as density of the rationals and irrationals. They demonstrate an understanding of one-to-one correspondence and its application to infinite sets and the notion of cardinality of infinite sets. Students will show an understanding of mathematical rigour and argument in solutions.

Chaos, Fractals; Geometry, Topology

In this unit, students will use a number of techniques to solve mathematical problems. They will demonstrate an understanding of properties of chaotic behaviour in dynamical system and the mathematical modelling properties of fractal sets. They will apply complex numbers to the analysis of fractal sets. Students use algebraic and discrete mathematical ideas to establish geometric properties and extend students' knowledge of geometric and algebraic notions to higher dimensions, non-Euclidian geometries, and topology. They will develop an understanding of mathematical rigour and argument in solutions.

Assessment

The identification of criteria within the achievement standards and assessment tasks types and weightings provide a common and agreed basis for the collection of evidence of student achievement.

Assessment Criteria (the dimensions of quality that teachers look for in evaluating student work) provide a common and agreed basis for judgement of performance against unit and course goals, within and across colleges. Over a course, teachers must use all these criteria to assess students' performance but are not required to use all criteria on each task. Assessment criteria are to be used holistically on a given task and in determining the unit grade.

Assessment Tasks elicit responses that demonstrate the degree to which students have achieved the goals of a unit based on the assessment criteria. The Common Curriculum Elements (CCE) is a guide to developing assessment tasks that promote a range of thinking skills (see Appendix C). It is highly desirable that assessment tasks engage students in demonstrating higher order thinking.

Rubrics are constructed for individual tasks, informing the assessment criteria relevant for a particular task, and can be used to assess a continuum that indicates levels of student performance against each criterion.

Assessment Criteria

Students will be assessed on the degree to which they demonstrate an understanding of:

- concepts and techniques
- reasoning and communications.

Assessment Task Types

Suggested tasks: <ul style="list-style-type: none"> • project/assignment • modelling projects • portfolio • journal • validation activity • presentation such as a pitch, poster, vodcast, interview • practical activity such as a demonstration • test/examination • online adaptive tasks/quiz 	
Weightings in T 1.0: No task be weighted more than 50% for a standard 1.0 unit.	

Additional Assessment Information

Requirements

- For a standard unit (1.0), students must complete a minimum of three assessment tasks and a maximum of five.
- Students should experience a variety of task types (test and non-test) and different modes of communication to demonstrate the Achievement Standards.
- Students are required to undertake at least one problem solving investigation task each semester. This task may be completed individually or collaboratively. They are required to plan, enquire into, and draw conclusions about key unit concepts. Students may respond in forms such as modelling projects, problem solving and practical activities.
- Assessment tasks for a standard (1.0) must be informed by the Achievement Standards.

Advice

- It is recommended that the total component of unsupervised tasks be no greater than 30%.
- For tasks completed in unsupervised conditions, schools need to have mechanisms to uphold academic integrity, for example, student declaration, plagiarism software, oral defence, interview, other validation tasks.

Achievement Standards

A Year 12 student in any unit is assessed using the Year 12 achievement standards. A Year 11 student in any unit is assessed using the Year 11 achievement standards. Year 12 achievement standards reflect higher expectations of student achievement compared to the Year 11 achievement standards. Years 11 and 12 achievement standards are differentiated by cognitive demand, the number of dimensions and the depth of inquiry.

An achievement standard cannot be used as a rubric for an individual assessment task. Assessment is the responsibility of the college. Student tasks may be assessed using rubrics or marking schemes devised by the college. A teacher may use the achievement standards to inform development of rubrics. The verbs used in achievement standards may be reflected in the rubric. In the context of combined Years 11 and 12 classes, it is best practice to have a distinct rubric for Years 11 and 12. These rubrics should be available for students prior to completion of an assessment task so that success criteria are clear.

Achievement Standards for Mathematics T Course – Year 11

	<i>A student who achieves an A grade typically</i>	<i>A student who achieves a B grade typically</i>	<i>A student who achieves a C grade typically</i>	<i>A student who achieves a D grade typically</i>	<i>A student who achieves an E grade typically</i>
Concepts and Techniques	<ul style="list-style-type: none"> critically applies mathematical concepts in a variety of complex contexts to routine and non-routine problems selects and applies advanced mathematical techniques to solve complex problems in a variety of contexts constructs, selects, and applies complex mathematical models to routine and non-routine problems in a variety of contexts uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts 	<ul style="list-style-type: none"> applies mathematical concepts in a variety of contexts to routine and non-routine problems selects and applies mathematical techniques to solve routine and non-routine problems in a variety of contexts selects and applies mathematical models to routine and non-routine problems to a variety of contexts uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts 	<ul style="list-style-type: none"> applies mathematical concepts in some contexts to routine and non-routine problems applies mathematical techniques to solve routine and non-routine problems in some contexts applies mathematical models to routine and non-routine problems in some contexts uses digital technologies appropriately to solve routine and non-routine problems in some contexts 	<ul style="list-style-type: none"> applies simple mathematical concepts in limited contexts to routine problems applies simple mathematical techniques to solve routine problems in limited contexts applies simple mathematical models to routine problems in limited contexts uses digital technologies appropriately to solve routine problems in limited contexts 	<ul style="list-style-type: none"> applies simple mathematical concepts in structured contexts uses simple mathematical techniques to solve routine problems in structured contexts demonstrates limited familiarity with mathematical models in structured contexts uses digital technologies to solve routine problems in structured contexts
Reasoning and Communications	<ul style="list-style-type: none"> represents complex mathematical concepts in numerical, graphical, and symbolic form in routine and non-routine problems in a variety of contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are succinct and well-reasoned, using appropriate and accurate language evaluates the reasonableness of solutions to routine and non-routine problems in a variety of contexts reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively evaluates the potential of Mathematics to generate knowledge in the public good 	<ul style="list-style-type: none"> represents mathematical concepts in numerical, graphical, and symbolic form in routine and non-routine problems a variety of contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, which are clear and reasoned, using appropriate and accurate language analyses the reasonableness of solutions to routine and non-routine problems reflects on their own thinking and analyses planning, time management, use of appropriate strategies to work independently and collaboratively analyses the potential of Mathematics to generate knowledge in the public good 	<ul style="list-style-type: none"> represents mathematical concepts in numerical, graphical and symbolic form to some routine and some non-routine problems in some contexts communicates mathematical judgements and arguments in oral, written and/or multimodal forms, using appropriate and accurate language explains the reasonableness of solutions to some routine and non-routine problems reflects on their own thinking and explains planning, time management, use of appropriate strategies to work independently and collaboratively explains the potential of Mathematics to generate knowledge in the public good 	<ul style="list-style-type: none"> represents simple mathematical concepts in numerical, graphical or symbolic form in routine problems in limited contexts communicates simple mathematical judgements or arguments in oral, written and/or multimodal forms, with some use of appropriate language describes the appropriateness of solutions to routine problems reflects on their own thinking with some reference to planning, time management, use of appropriate strategies to work independently and collaboratively describes the potential of Mathematics to generate knowledge in the public good 	<ul style="list-style-type: none"> represents simple mathematical concepts in numerical, graphical or symbolic form in structured contexts communicates simple mathematical information in oral, written and/or multimodal forms, with limited use of appropriate language identifies solutions to routine problems in structured contexts reflects on their own thinking with little or no reference to planning, time management, use of appropriate strategies to work independently and collaboratively identifies some ways in which Mathematics is used to generate knowledge in the public good

Achievement Standards for Mathematics T Course – Year 12

	<i>A student who achieves an A grade typically</i>	<i>A student who achieves a B grade typically</i>	<i>A student who achieves a C grade typically</i>	<i>A student who achieves a D grade typically</i>	<i>A student who achieves an E grade typically</i>
Concepts and Techniques	<ul style="list-style-type: none"> critically and creatively applies mathematical concepts in a variety of complex contexts to routine and non-routine problems synthesises information to select and apply mathematical techniques to solve complex problems in a variety of contexts constructs, selects, and applies mathematical models to a variety of contexts in routine and non-routine problems uses digital technologies efficiently to solve routine and non-routine problems in a variety of contexts 	<ul style="list-style-type: none"> critically applies mathematical concepts in a variety of contexts to routine and non-routine problems analyses information to select and apply mathematical techniques to solve routine and non-routine problems in a variety of contexts selects and applies mathematical models to routine and non-routine problems in a variety of contexts uses digital technologies effectively to solve routine and non-routine problems in a variety of contexts 	<ul style="list-style-type: none"> applies mathematical concepts in some contexts to routine and non-routine problems selects and applies mathematical techniques to solve routine and some non-routine problems in some contexts applies mathematical models to routine and non-routine problems in some contexts uses digital technologies appropriately to solve routine and non-routine problems in a variety of contexts 	<ul style="list-style-type: none"> applies simple mathematical concepts in limited contexts to routine problems applies simple mathematical techniques to solve routine problems in limited contexts applies simple mathematical models to routine problems in limited contexts uses digital technologies appropriately to solve routine problems in limited contexts 	<ul style="list-style-type: none"> applies simple mathematical concepts in structured contexts uses simple mathematical techniques to solve routine problems in structured contexts demonstrates limited familiarity with mathematical models to solve routine problems in structured contexts uses digital technologies to solve routine problems in structured contexts
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Cryptography and Infinite Cardinals

Value: 1.0

Unit Description

This unit contains two modules: 'Number Contemplation and Cryptography', and 'A Hierarchy of Infinite Cardinals'. In this unit, students gain an understanding of mathematical patterns and their application to cryptography, and the cardinality of infinite sets. They expand their understanding of real number systems, including concepts such as density of rationals and irrationals. Students will also learn how to demonstrate mathematical rigour and argument in solutions.

Specific Unit Goals

This unit should enable students to:

- understand the concepts and techniques in number patterns and cryptography, and infinite cardinals
- apply reasoning skills and solve problems in number patterns and cryptography, and infinite cardinals
- demonstrate mathematical rigour and argument to communicate solutions.

Content Descriptions

All knowledge, understanding and skills below must be delivered:

Concepts and Techniques

- solve problems and prove results using the pigeon-hole principle
- analyse number sets to identify mathematical patterns including further properties of the Fibonacci sequence and proof of Binet's formula
- apply mathematical patterns to generate sophisticated cryptography through:
 - Prime numbers and their properties
 - Modular arithmetic
 - Twin Prime Conjecture, Goldbach Conjecture and Fermat's Last Theorem
 - Fermat's Little Theorem proof
 - Developing cryptographic algorithms
- prove that RSA public key cryptography always works
- investigate real number systems including concepts such as density of rationals and irrationals
- investigate infinite sets and how they are different through exploring one-to-one correspondence and countability of infinite sets
- determine the cardinality of the set of real numbers and power sets, including Cantors diagonalisation proof, Russel's Paradox, and geometric argument.

Reasoning and Communication

- use elegant and efficient techniques to solve mathematical problems
- use conventions of mathematical rigour and argument to communicate solutions
- represent mathematical concepts in numerical, graphical, and symbolic form in routine and non-routine problems in a variety of contexts
- reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively
- evaluates the potential of Mathematics to generate knowledge in the public good.

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 8-10.

Chaos, Fractals; Geometry, Topology

Value: 1.0

Unit Description

In this unit, students will use a number of techniques to solve mathematical problems. They will demonstrate an understanding of properties of chaotic behaviour in dynamical system and the mathematical modelling properties of fractal sets. They will apply complex numbers to the analysis of fractal sets. Students use algebraic and discrete mathematical ideas to establish geometric properties and extend students' knowledge of geometric and algebraic notions to higher dimensions, non-Euclidian geometries, and topology. They will develop an understanding of mathematical rigour and argument in solutions.

Specific Unit Goals

This unit should enable students to:

- understand the concepts and techniques in chaotic behaviour, fractal sets, geometry, and topology
- apply reasoning skills and solve problems in chaotic behaviour, fractal sets, geometry, and topology
- demonstrate mathematical rigour and argument to communicate solutions.

Content Descriptions

All knowledge, understanding and skills below must be delivered:

Concepts and Techniques

- investigate chaotic behaviour in iterative dynamical systems such as population growth and Logistic equation
- model Fractals using iterated function systems and the chaos game
- investigate self-similar objects and the application of complex numbers to fractals to a number of natural and artificial phenomena including Julia and Mandelbrot sets
- analyse the dimensions of fractals and the application of scaling parameters
- apply geometric and algebraic notions to prove Euler's formula and that there are only five platonic solids
- investigate higher dimensions, non-Euclidian geometries, and topology through topics such as:
 - Aperiodic Tilings of the Plane
 - Spherical and Hyperbolic Geometries
 - Visualising the Fourth Dimension
 - Basic notions of Topology
 - Two Dimensional Surfaces
 - Knots and Links
 - Fixed Point Theorems

Reasoning and communication

- use elegant and efficient techniques to solve mathematical problems
- use conventions of mathematical rigour and argument to communicate solutions
- represent mathematical concepts in numerical, graphical, and symbolic form in routine and non-routine problems in a variety of contexts
- reflects with insight on their own thinking and that of others and evaluates planning, time management, use of appropriate strategies to work independently and collaboratively
- evaluates the potential of Mathematics to generate knowledge in the public good
- use elegant and efficient techniques to solve mathematical problems.

A guide to reading and implementing content descriptions

Content descriptions specify the knowledge, understanding and skills that students are expected to learn and that teachers are expected to teach. Teachers are required to develop a program of learning that allows students to demonstrate all the content descriptions. The lens which the teacher uses to demonstrate the content descriptions may be either guided through provision of electives within each unit or determined by the teacher when developing their program of learning.

A program of learning is what a college provides to implement the course for a subject. It is at the discretion of the teacher to emphasis some content descriptions over others. The teacher may teach additional (not listed) content provided it meets the specific unit goals. This will be informed by the student needs and interests.

Assessment

Refer to pages 8-10.

Appendix A – Implementation Guidelines

Available course patterns

A standard 1.0 value unit is delivered over at least 55 hours. To be awarded a course, students must complete at least the minimum units over the whole minor, major, major/minor, or double major course.

Course	Number of standard units to meet course requirements
Minor	Minimum of 2 units

Units in this course can be delivered in any order.

Co-requisites for the course

Students must be enrolled in BSSS Specialist Mathematics Course in their home college to be eligible for this H Course.

Duplication of Content Rules

Students cannot be given credit towards the requirements for a Senior Secondary Certificate for a unit that significantly duplicates content in a unit studied in another course. The responsibility for preventing undesirable overlap of content studied by a student rests with the principal and the teacher delivering the course. Students will only be given credit for covering the content once.

Guidelines for Delivery

Program of Learning

A program of learning is what a school provides to implement the course for a subject. This meets the requirements for context, scope and sequence set out in the Board endorsed course. Students follow programs of learning in a college as part of their senior secondary studies. The detail, design, and layout of a program of learning are a college decision.

The program of learning must be documented to show the planned learning activities and experiences that meet the needs of particular groups of students, taking into account their interests, prior knowledge, abilities, and backgrounds. The program of learning is a record of the learning experiences that enable students to achieve the knowledge, understanding and skills of the content descriptions. There is no requirement to submit a program of learning to the OBSSS for approval. The Principal will need to sign off at the end of Year 12 that courses have been delivered as accredited.

Content Descriptions

Are all content descriptions of equal importance? No. It depends on the focus of study. Teachers can customise their program of learning to meet their own students' needs, adding additional content descriptions if desired or emphasising some over others. A teacher must balance student needs with their responsibility to teach all content descriptions. It is mandatory that teachers address all content descriptions and that students engage with all content descriptions.

Half standard 0.5 units

Half standard units appear on the course adoption form but are not explicitly documented in courses. It is at the discretion of the college principal to split a standard 1.0 unit into two half standard 0.5 units. Colleges are required to adopt the half standard 0.5 units. However, colleges are not required to submit explicit documentation outlining their half standard 0.5 units to the BSSS. Colleges must assess students using the half standard 0.5 assessment task weightings outlined in the framework. It is the responsibility of the college principal to ensure that all content is delivered in units approved by the Board.

Moderation

Moderation is a system designed and implemented to:

- provide comparability in the system of school-based assessment
- form the basis for valid and reliable assessment in senior secondary schools
- involve the ACT Board of Senior Secondary Studies and colleges in cooperation and partnership
- maintain the quality of school-based assessment and the credibility, validity, and acceptability of Board certificates.

Moderation commences within individual colleges. Teachers develop assessment programs and instruments, apply assessment criteria, and allocate Unit Grades, according to the relevant Framework. Teachers within course teaching groups conduct consensus discussions to moderate marking or grading of individual assessment instruments and Unit Grade decisions.

The Moderation Model

Moderation within the ACT encompasses structured, consensus-based peer review of Unit Grades for all accredited courses over two Moderation Days. In addition to Moderation Days, there is statistical moderation of course scores, including small group procedures, for T courses.

Moderation by Structured, Consensus-based Peer Review

Consensus-based peer review involves the review of student work against system wide criteria and standards and the validation of Unit Grades. This is done by matching student performance with the criteria and standards outlined in the Achievement Standards, as stated in the Framework. Advice is then given to colleges to assist teachers with, or confirm, their judgments. In addition, feedback is given on the construction of assessment instruments.

Preparation for Structured, Consensus-based Peer Review

Each year, teachers of Year 11 are asked to retain originals or copies of student work completed in Semester 2. Similarly, teachers of a Year 12 class should retain originals or copies of student work completed in Semester 1. Assessment and other documentation required by the Office of the Board of Senior Secondary Studies should also be kept. Year 11 work from Semester 2 of the previous year is presented for review at Moderation Day 1 in March, and Year 12 work from Semester 1 is presented for review at Moderation Day 2 in August.

In the lead up to Moderation Day, a College Course Presentation (comprised of a document folder and a set of student portfolios) is prepared for each A, T and M course/units offered by the school and is sent into the Office of the Board of Senior Secondary Studies.

The College Course Presentation

The package of materials (College Course Presentation) presented by a college for review on Moderation Days in each course area will comprise the following:

- a folder containing supporting documentation as requested by the Office of the Board through memoranda to colleges, including marking schemes and rubrics for each assessment item
- a set of student portfolios containing marked and/or graded written and non-written assessment responses and completed criteria and standards feedback forms. Evidence of all assessment responses on which the Unit Grade decision has been made is to be included in the student review portfolios.

Specific requirements for subject areas and types of evidence to be presented for each Moderation Day will be outlined by the Board Secretariat through the *Requirements for Moderation Memoranda* and Information Papers.

Visual evidence for judgements made about practical performances

It is a requirement that schools' judgements of standards to practical performances (A/T/M) be supported by visual evidence (still photos or video).

The photographic evidence submitted must be drawn from practical skills performed as part of the assessment process.

Teachers should consult the BSSS guidelines at:

http://www.bsss.act.edu.au/grade_moderation/moderation_information_for_teachers

for current information regarding all moderation requirements including subject specific and photographic evidence.

for current information regarding all moderation requirements including subject specific and photographic evidence.

Appendix B – Course Developers

Name	College
Chris Wetherell	Australian Mathematics Trust
Lisa Walker	Narrabundah College
Griffith Ware	ANU
Prof John Hutchinson	ANU
Nazim Khan	St Mary McKillop College

Appendix C – Common Curriculum Elements

Common curriculum elements assist in the development of high-quality assessment tasks by encouraging breadth and depth and discrimination in levels of achievement.

Organisers	Elements	Examples
create, compose, and apply	apply	ideas and procedures in unfamiliar situations, content, and processes in non-routine settings
	compose	oral, written, and multimodal texts, music, visual images, responses to complex topics, new outcomes
	represent	images, symbols, or signs
	create	creative thinking to identify areas for change, growth, and innovation, recognise opportunities, experiment to achieve innovative solutions, construct objects, imagine alternatives
	manipulate	images, text, data, points of view
analyse, synthesise, and evaluate	justify	arguments, points of view, phenomena, choices
	hypothesise	statement/theory that can be tested by data
	extrapolate	trends, cause/effect, impact of a decision
	predict	data, trends, inferences
	evaluate	text, images, points of view, solutions, phenomenon, graphics
	test	validity of assumptions, ideas, procedures, strategies
	argue	trends, cause/effect, strengths and weaknesses
	reflect	on strengths and weaknesses
	synthesise	data and knowledge, points of view from several sources
	analyse	text, images, graphs, data, points of view
	examine	data, visual images, arguments, points of view
	investigate	issues, problems
organise, sequence, and explain	sequence	text, data, relationships, arguments, patterns
	visualise	trends, futures, patterns, cause, and effect
	compare/contrast	data, visual images, arguments, points of view
	discuss	issues, data, relationships, choices/options
	interpret	symbols, text, images, graphs
	explain	explicit/implicit assumptions, bias, themes/arguments, cause/effect, strengths/weaknesses
	translate	data, visual images, arguments, points of view
	assess	probabilities, choices/options
	select	main points, words, ideas in text
identify, summarise and plan	reproduce	information, data, words, images, graphics
	respond	data, visual images, arguments, points of view
	relate	events, processes, situations
	demonstrate	probabilities, choices/options
	describe	data, visual images, arguments, points of view
	plan	strategies, ideas in text, arguments
	classify	information, data, words, images
	identify	spatial relationships, patterns, interrelationships
	summarise	main points, words, ideas in text, review, draft and edit

Appendix D – Glossary of Verbs

Verbs	Definition
Analyse	Consider in detail for the purpose of finding meaning or relationships, and identifying patterns, similarities and differences
Apply	Use, utilise or employ in a particular situation
Argue	Give reasons for or against something
Assess	Make a Judgement about the value of
Classify	Arrange into named categories in order to sort, group or identify
Compare	Estimate, measure or note how things are similar or dissimilar
Compose	The activity that occurs when students produce written, spoken, or visual texts
Contrast	Compare in such a way as to emphasise differences
Create	Bring into existence, to originate
Demonstrate	Give a practical exhibition an explanation
Describe	Give an account of characteristics or features
Discuss	Talk or write about a topic, taking into account different issues or ideas
Evaluate	Examine and judge the merit or significance of something
Examine	Determine the nature or condition of
Explain	Provide additional information that demonstrates understanding of reasoning and /or application
Extrapolate	Infer from what is known
Hypothesise	Put forward a supposition or conjecture to account for certain facts and used as a basis for further investigation by which it may be proved or disproved
Identify	Recognise and name
Interpret	Draw meaning from
Investigate	Planning, inquiry into and drawing conclusions about
Justify	Show how argument or conclusion is right or reasonable
Manipulate	Adapt or change
Plan	Strategize, develop a series of steps, processes
Predict	Suggest what might happen in the future or as a consequence of something
Reflect	The thought process by which students develop an understanding and appreciation of their own learning. This process draws on both cognitive and affective experience
Relate	Tell or report about happenings, events, or circumstances
Represent	Use words, images, symbols or signs to convey meaning
Reproduce	Copy or make close imitation
Respond	React to a person or text
Select	Choose in preference to another or others
Sequence	Arrange in order
Summarise	Give a brief statement of the main points
Synthesise	Combine elements (information/ideas/components) into a coherent whole
Test	Examine qualities or abilities
Translate	Express in another language or form, or in simpler terms
Visualise	The ability to decode, interpret, create, question, challenge and evaluate texts that communicate with visual images as well as, or rather than, words

Appendix E – Glossary for ACT Senior Secondary Curriculum

Courses will detail what teachers are expected to teach and students are expected to learn for year 11 and 12. They will describe the knowledge, understanding and skills that students will be expected to develop for each learning area across the years of schooling.

Learning areas are broad areas of the curriculum, including English, mathematics, science, the arts, languages, health, and physical education.

A **subject** is a discrete area of study that is part of a learning area. There may be one or more subjects in a single learning area.

Frameworks are system documents for Years 11 and 12 which provide the basis for the development and accreditation of any course within a designated learning area. In addition, frameworks provide a common basis for assessment, moderation and reporting of student outcomes in courses based on the framework.

The **course** sets out the requirements for the implementation of a subject. Key elements of a course include the rationale, goals, content descriptions, assessment, and achievement standards as designated by the framework.

BSSS courses will be organised into units. A unit is a distinct focus of study within a course. A standard 1.0 unit is delivered for a minimum of 55 hours generally over one semester.

Core units are foundational units that provide students with the breadth of the subject.

Additional units are avenues of learning that cannot be provided for within the four core 1.0 standard units by an adjustment to the program of learning.

A **negotiated study unit** makes provision for students, classes, groups, or individuals to negotiate the program of learning based on the specific unit goals, content descriptions, assessment, and achievement standards of the course.

An **elective** is a lens for demonstrating the content descriptions within a standard 1.0

A **lens** is a particular focus or viewpoint within a broader study.

Content descriptions refer to the subject-based knowledge, understanding and skills to be taught and learned.

A **program of learning** is what a college develops to implement the course for a subject and to ensure that the content descriptions are taught and learned.

Achievement standards provide an indication of typical performance at five different levels (corresponding to grades A to E) following completion of study of senior secondary course content for units in a subject.

ACT senior secondary system **curriculum** comprises all BSSS approved courses of study.