

ANU EXTENSION



SPECIALIST MATHEMATICS - ANU  
H COURSE



**H Course**

**College: The Australian National University Extension**

**Course: Specialist Mathematics - ANU**

**Classification: H**

Course Code

Unit Title(s)	Value (1.0)	Length	Unit Codes
Number Contemplation and Cryptography	0.5	3 Q	
A Hierarchy of Infinite Cardinals	0.5	3 Q	
Geometry and Topology	0.5	3 Q	
Chaos and Fractals	0.5	3 Q	
Cryptography; Infinite Cardinals	1.0	1.5 Q	
Chaos, Fractals; Geometry, Topology	1.0	1.5 Q	
Geometry, Topology; Cryptography	1.0	1.5 Q	
Infinite Cardinals; Chaos, Fractals	1.0	1.5 Q	

Dates of Course Recognition: From 2016 To 2020

**H Classification:** The course and units named above are consistent with H course policy and are signed on behalf of the BSSS.

Course Development Coordinator: / /	Panel Chair: / /
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## Course Name: Specialist Mathematics ANU

### Course Classification: H

### Course Developers

Name	Qualifications
Lisa Walker	BSc Mathematics ANU, Post grad Dip Ed UNSW, BA UNSW
Griffith Ware	PhD (MSI) ANU , BSc (Honors in Mathematics) ANU
Prof John Hutchinson	PhD Stanford, BSc (Honors in Mathematics) UNSW
Neil Montgomery	BSc, Grad Dip Ed (Sydney)

### Course Length and Composition

*The definition of a unit and hour requirements for a standard unit and course duration, as outlined in policies 3.2.9 Unit, 3.2.9.1 Unit Values and 3.2.8.3 Course Duration (2015 BSSS Policy and Procedures Manual), apply to H courses.*

#### Name and Number of Units Submitted and the Length of the Units expressed as a Value

Unit Title	Unit Value
Number Contemplation and Cryptography	0.5
A Hierarchy of Infinite Cardinals	0.5
Geometry and Topology	0.5
Chaos and Fractals	0.5
Cryptography; Infinite Cardinals	1.0
Chaos, Fractals; Geometry, Topology	1.0
Geometry, Topology; Cryptography	1.0
Infinite Cardinals; Chaos, Fractals	1.0

#### Available Course Patterns

Course	Minimum number of hours per course	Number of standard 1.0 value units to meet course requirements
Minor	110 hours	2 units of 55 hours

The ANU Extension Specialist Mathematics H Course comprises four modules, each being of 0.5 unit value:

- a. Number Contemplation and Cryptography,
- b. A Hierarchy of Infinite Cardinals,
- c. Geometry and Topology, and
- d. Chaos and Fractals.

Each cohort of students taking the full course will study these four modules over a two-year period. The modules may be taught in the order (a)-(b)-(d)-(c), or (c)-(a)-(b)-(d). The different combinations of these modules constitute different unit names.

1. Cryptography; Infinite Cardinals,
2. Chaos, Fractals; Geometry, Topology,
3. Geometry, Topology; Cryptography, and
4. Infinite Cardinals; Chaos, Fractals.

Students cannot use any of the models more than once to build their minor in Specialist Mathematics ANU.

## **Implementation Guidelines**

### **Arrangements for continuing students**

Students completing the Numbers and Cryptography; Infinity (10603) in 2015 in the previous Specialist Mathematics H course (1056) may combine that unit with the Chaos, Fractals; Geometry, Topology unit from this course in 2016 to form a minor.

### **Prerequisites or co requisites home college course/s**

Students must be enrolled in a Specialist Mathematics major minor at their home school/college concurrently with this course. Students must be granted entry to ANU Extension.

### **Contribution towards an ATAR**

Students can count up to 2 H courses to a maximum weight of 1.2 (equivalent to 2 minors) out of the required 3.6 in the calculation of the ATAR.

A maximum of 4 standard units from H courses can contribute to the minimum requirements for an ACT Senior Secondary Certificate and Tertiary Entrance Statement.

A minor in Specialist Mathematics ANU can be combined with a major minor Specialist Mathematics from a student's home college to form a double major.

### **Reporting of H courses on the ACT Senior Secondary Certificate**

A T classified major minor and H minor in the same subject are equivalent to a double major course type.

H courses are reported separately on the Senior Secondary Certificate and Tertiary Entrance Statement. If a student has not completed enough H units to form a course, the units may be reported under the relevant home college course and contribute to the course type but not the course score. (*Refer section 9 University Programs in 2015 Policies and Procedures Manual*)

## Goals

The goal is to introduce students to contemporary mainstream 20th and 21st century mathematics.

Students will investigate some very exciting and useful modern mathematics and get a feeling for "what mathematics is all about". The mathematics which students will see in this course is usually not covered until higher level courses in second or third year at University. Naturally, it is then studied in much greater depth. Students will study carefully chosen parts and representative examples from various areas of mathematics which illustrate important and general key concepts. In the process students will have the opportunity to gain a real understanding and feeling for the beauty, utility and breadth of mathematics.

## Student Group

Students apply to ANU for entry to this course and suitable applicants are selected at the beginning of their Year 11 year through a selection process. This process may include some or all of; a selection test, evidence provided of past academic successes, school/college recommendation.

Students must be enrolled in a Specialist mathematics major minor at an ACT school/college.

A student who achieves a satisfactory standard in this course will be made an early offer of entry to the ANU. The offer will be for the BSc or BSc (Advanced) Hons or PhB course depending on the level of achievement. A pass in this course is not a guarantee of an early offer. Early offers are decided by the ANU Dean of Sciences. Students who are made early offers will also be awarded a six point credit towards an ANU undergraduate course.

## Assessment

There will be 3 – 5 summative assessment items for a 1.0 point unit and 2 – 3 for a 0.5 point unit.

<b>Task Type</b>	<b>Weighting</b>	
<b>Tests:</b> For example: <ul style="list-style-type: none"> <li>- Multiple Choice</li> <li>- Short Answer</li> <li>- Extended Questions</li> </ul>	40-75%	
<b>Non-Test Tasks (in class)</b> For example: <ul style="list-style-type: none"> <li>- Modelling</li> <li>- Investigations</li> <li>- Problem solving</li> <li>- Journals</li> <li>- Portfolios</li> <li>- Presentations</li> <li>- Practical activities</li> </ul>	0 - 60%	25-60%
<b>Take Home Tasks:</b> For example: <ul style="list-style-type: none"> <li>- modelling</li> <li>- investigations</li> <li>- portfolios</li> <li>- practical activities</li> </ul>	0 – 30%	

## Moderation

### 9.2.2 Moderation of H courses (2015 BSSS Policies and Procedures Manual)

A Year 11 review portfolio will be prepared in December, after the end of the first 1.0 point unit, for Moderation Day 1 the following year.

A Year 12 review portfolio will be prepared by Week 3, Term 4 following the completion of the Year 12 unit at the end of Term 3.

Review portfolios will present the work of two students at different grade levels.

Grades in H courses are not subject to moderation.

## Bibliography

### Books

[1] Burger E B & Starbird M, *The Heart of Mathematics: An invitation to Effective Thinking (fourth, third, or second edition)*, John Wiley & Sons, Inc., Hoboken, NJ, 2013, 2010, 2005.

**Other Reference Books**

[2] Tannenbaum, P, *Excursions in Modern Mathematics (fifth edition)*, Pearson Prentice Hall, 2004.

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**Audio-visual Material****CD ROMS**

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### Prerequisites

Students must be granted entry to ANU Extension, and have achieved at least an ANU pass grade in previous ANU Extension subjects they have taken.

### Specific Unit Goals

This unit should enable students to:

- Use a number of techniques to solve mathematical problems;
- Demonstrate an understanding of mathematical patterns and their application to cryptography; and
- Show an understanding of mathematical rigor and argument in solutions.

### Content

- Conundrums that evoke effective thinking
- Review of the 'Pigeon Hole Principle'
- Numerical patterns in nature
  - Review and establish further properties of the Fibonacci sequence
  - Proof of Binet's formula
- RSA public key cryptography
  - Review prime numbers and their properties
  - Review modular arithmetic
  - Discuss the Twin Prime Conjecture, Goldbach Conjecture and Fermat's Last Theorem
  - Prove Fermat's Little Theorem
  - Develop the algorithm for RSA public key cryptography
  - Proof that RSA public key cryptography always works

### Teaching and Learning Strategies

Lectures and tutorials

Presentations from academics with relevant expertise

Guided discussions

Group investigations

Student presentations

Student initiated investigations and research

Use of interactive computer software and simulations

Online resources

## Assessment

There will be 4 – 6 summative assessment items.

T course:

Criteria /Task	Exam	Ongoing Assignment	Project
Knowledge	✓	✓	✓
Application	✓	✓	✓
Communication	✓	✓	✓
Argument	✓	✓	✓
Appropriate Technology	✓	✓	✓
Weighting	50 - 70%	0 – 40%	0 - 40%

## Specific Unit Resources

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## Specific Unit Goals

This unit should enable students to:

- Use a number of techniques to solve mathematical problems;
- Demonstrate an understanding of the real number system including concepts such as density of the rationals and irrationals;
- Demonstrate an understanding of one-to-one correspondence and its application to infinite sets;
- Demonstrate an understanding of the notion of cardinality of infinite sets; and
- Show an understanding of mathematical rigor and argument in solutions.

## Content

- The real number line
  - Review of irrational numbers
  - Investigate the density of the rational and irrational numbers
- One-to-one correspondence between infinite sets
- Countability of infinite sets
  - Including the sets of even integers, odd integers, all integers and rational numbers
- Cardinality of the set of real numbers
  - Include Cantor's diagonalization proof
- Power sets and cardinality
  - Include Russel's Paradox
- Cardinality by geometric argument
- (If time) Further analysis of the properties of the real number system
  - for example the notion of completeness

## Teaching and Learning Strategies

Lectures and tutorials

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Online resources

## Assessment

There will be 4 – 6 summative assessment items.

T course:

Criteria /Task	Exam	Ongoing Assignment	Project
Knowledge	✓	✓	✓
Application	✓	✓	✓
Communication	✓	✓	✓
Argument	✓	✓	✓
Appropriate Technology	✓	✓	✓
Weighting	50 - 70%	0 – 40%	0 - 40%

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## Specific Unit Goals

This unit should enable students to:

- Use a number of techniques to solve mathematical problems;
- Use algebraic and discrete mathematical ideas to establish geometric properties;
- Extend students' knowledge of geometric and algebraic notions to higher dimensions, non-Euclidian geometries and topology; and
- Show an understanding of mathematical rigor and argument in solutions.

## Content

- The 5 Platonic solids
  - Prove Euler's formula
  - Prove that there are no other Platonic solids using Euler's formula
- A selection of topics from;
  - Aperiodic Tilings of the Plane
  - Spherical and Hyperbolic Geometries
  - Visualising the Fourth Dimension
  - Basic notions of Topology
  - Two Dimensional Surfaces
  - Knots and Links
  - Fixed Point Theorems

## Teaching and Learning Strategies

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Group investigations

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Online resources

## Assessment

There will be 4 – 6 summative assessment items.

T course:

Criteria /Task	Exam	Ongoing Assignment	Project
Knowledge	✓	✓	✓
Application	✓	✓	✓
Communication	✓	✓	✓
Argument	✓	✓	✓
Appropriate Technology	✓	✓	✓
Weighting	50 - 70%	0 – 40%	0 - 40%

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## Specific Unit Goals

This unit should enable students to:

- Use a number of techniques to solve mathematical problems;
- Demonstrate an understanding of properties of chaotic behaviour in dynamical systems;
- Demonstrate an understanding of the mathematical modelling properties of fractal sets;
- Apply complex numbers to the analysis of fractal sets; and
- Show an understanding of mathematical rigor and argument in solutions.

## Content

- Iterative Dynamical Systems
  - Population growth, Logistic equation and extensions
- Generating Fractals through iterative processes
  - Self similarity in nature over a range of scales
  - Iterated Functions Systems
  - The Chaos Game
- Julia and Mandelbrot sets
  - Application of Complex Numbers
- Deterministic Chaos
- Chaotic behaviour arising from deterministic systems
- Dimensions of Fractals
  - Dimensions via Scaling Arguments

## Teaching and Learning Strategies

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Student presentations

Student initiated investigations and research

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Online resources

## Assessment

There will be 4 – 6 summative assessment items.

T course:

Criteria /Task	Exam	Ongoing Assignment	Project
Knowledge	✓	✓	✓
Application	✓	✓	✓
Communication	✓	✓	✓
Argument	✓	✓	✓
Appropriate Technology	✓	✓	✓
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## Specific Unit Goals

This unit should enable students to:

- Use a number of techniques to solve mathematical problems;
- Demonstrate an understanding of mathematical patterns and their application to cryptography;
- Demonstrate an understanding of the real number system including concepts such as density of the rationals and irrationals;
- Demonstrate an understanding of one-to-one correspondence and its application to infinite sets;
- Demonstrate an understanding of the notion of cardinality of infinite sets; and
- Show an understanding of mathematical rigor and argument in solutions.

## Content

- Conundrums that evoke effective thinking
- Review of the 'Pigeon Hole Principle'
- Numerical patterns in nature
  - Review and establish further properties of the Fibonacci sequence
  - Proof of Binet's formula
- RSA public key cryptography
  - Review prime numbers and their properties
  - Review modular arithmetic
  - Discuss the Twin Prime Conjecture, Goldbach Conjecture and Fermat's Last Theorem
  - Prove Fermat's Little Theorem
  - Develop the algorithm for RSA public key cryptography
  - Proof that RSA public key cryptography always works
- The real number line
  - Review of irrational numbers
  - Investigate the density of the rational and irrational numbers
- One-to-one correspondence between infinite sets
- Countability of infinite sets
  - Including the sets of even integers, odd integers, all integers and rational numbers
- Cardinality of the set of real numbers
  - Include Cantor's diagonalization proof
- Power sets and cardinality
  - Include Russel's Paradox
- Cardinality by geometric argument
- (If time) Further analysis of the properties of the real number system
  - for example the notion of completeness

## Teaching and Learning Strategies

Lectures and tutorials  
Presentations from academics with relevant expertise  
Guided discussions  
Group investigations  
Student presentations  
Student initiated investigations and research  
Use of interactive computer software and simulations  
Online resources

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There will be 4 – 6 summative assessment items.

**T course:**

Criteria /Task	Exam	Ongoing Assignment	Project
Knowledge	✓	✓	✓
Application	✓	✓	✓
Communication	✓	✓	✓
Argument	✓	✓	✓
Appropriate Technology	✓	✓	✓
Weighting	50 - 70%	0 – 40%	0 - 40%

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## Specific Unit Goals

This unit should enable students to:

- Use a number of techniques to solve mathematical problems;
- Demonstrate an understanding of properties of chaotic behaviour in dynamical systems;
- Demonstrate an understanding of the mathematical modelling properties of fractal sets;
- Apply complex numbers to the analysis of fractal sets;
- Use algebraic and discrete mathematical ideas to establish geometric properties;
- Extend students' knowledge of geometric and algebraic notions to higher dimensions, non-Euclidian geometries and topology; and
- Show an understanding of mathematical rigor and argument in solutions.

## Content

- Iterative Dynamical Systems
  - Population growth, Logistic equation and extensions
- Generating Fractals through iterative processes
  - Self similarity in nature over a range of scales
  - Iterated Functions Systems
  - The Chaos Game
- Julia and Mandelbrot sets
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- Deterministic Chaos
- Chaotic behaviour arising from deterministic systems
- Dimensions of Fractals
- Dimensions via Scaling Arguments
- The 5 Platonic solids
  - Prove Euler's formula
  - Prove that there are no other Platonic solids using Euler's formula
- A selection of topics from;
  - Aperiodic Tilings of the Plane
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- Demonstrate an understanding of one-to-one correspondence and its application to infinite sets;
- Demonstrate an understanding of the notion of cardinality of infinite sets;
- Demonstrate an understanding of properties of chaotic behaviour in dynamical systems;
- Demonstrate an understanding of the mathematical modelling properties of fractal sets;
- Apply complex numbers to the analysis of fractal sets; and
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## Content

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- Generating Fractals through iterative processes
  - Self similarity in nature over a range of scales
  - Iterated Functions Systems
  - The Chaos Game
- Julia and Mandelbrot sets
  - Application of Complex Numbers
- Deterministic Chaos
- Chaotic behaviour arising from deterministic systems
- Dimensions of Fractals

- Dimensions via Scaling Arguments

## Teaching and Learning Strategies

Lectures and tutorials  
 Presentations from academics with relevant expertise  
 Guided discussions  
 Group investigations  
 Student presentations  
 Student initiated investigations and research  
 Use of interactive computer software and simulations

## Assessment

There will be 4 – 6 summative assessment items.

**T course:**

Criteria /Task	Exam	Ongoing Assignment	Project
Knowledge	✓	✓	✓
Application	✓	✓	✓
Communication	✓	✓	✓
Argument	✓	✓	✓
Appropriate Technology	✓	✓	✓
Weighting	50 - 70%	0 – 40%	0 - 40%

## Specific Unit Resources

### Books

[1] Burger E B & Starbird M, *The Heart of Mathematics: An invitation to Effective Thinking (fourth, third, or second edition)*, John Wiley & Sons, Inc., Hoboken, NJ, 2013, 2010, 2005.

### Other Reference Books

[2] Tannenbaum, P, *Excursions in Modern Mathematics (fifth edition)*, Pearson Prentice Hall, 2004.

[3] COMAP consortium, *For all Practical Purposes, Mathematical Literacy in Today's World (sixth edition)*, Freeman and Co., NY, 2003.

### ANU Extension H Course Notes

[4] Hutchinson, J, *An Introduction to Contemporary Mathematics*.

**Journal Articles**

Listed at the end of each relevant chapter in [2] and [3].

**Audio-visual Material****CD ROMS**

Supplied with *The Heart of Mathematics*.

**Web sites**

<http://www.heartofmath.com/>

Listed at the end of each relevant chapter in [3].

These were accurate at the time of publication.